Ultra-Fast Asynchronous Rumor Spreading

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Push Protocol (Synchronous)

Demers, Gealy, Greene, Hauser, Irish, Larson, Manning, Shenker, Sturgis, Swinehart, Terry, Woods'87

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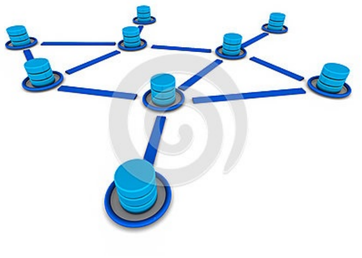
Spread Time: the first time everyone knows the rumour.

Application: distributed computing



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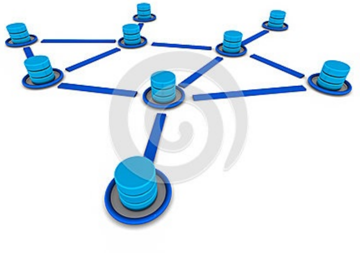
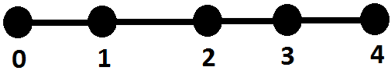


Rumour spreading advantages:

- ✓ Simplicity, locality, no memory
- ✓ Scalability, reasonable link loads
- ✓ Robustness

Example: a path



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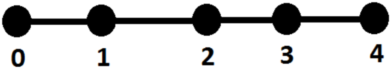

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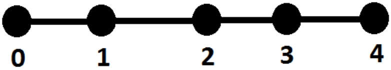

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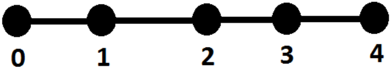

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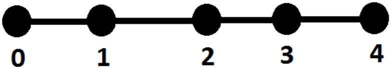

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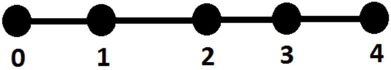

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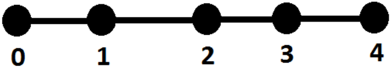

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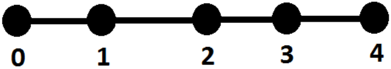

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&= 2n - 3
\end{aligned}$$

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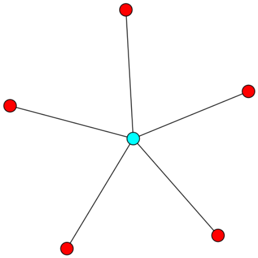

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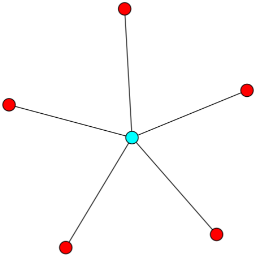

When $k+1$ vertices are informed and $n-1-k$ uninformed, after $\mathbb{E}[\text{Geo}(\frac{n-k-1}{n-1})] = \frac{n-1}{n-1-k}$ more rounds a new vertex will be informed.

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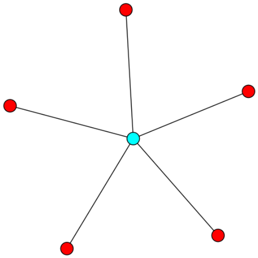

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$$\mathbb{E}[\text{Spread Time}] = \frac{n-1}{n-1} + \frac{n-1}{n-2} + \cdots + \frac{n-1}{2} + \frac{n-1}{1} \approx n \ln n$$

Improving the protocol

Uninformed vertices ask the informed ones...

Push-Pull Protocol (Synchronous)

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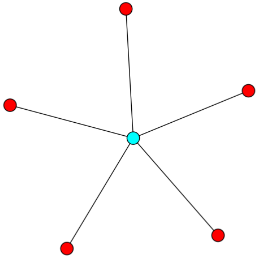

push protocol: $n \ln n$ rounds
push-pull protocol: 1 or 2 rounds

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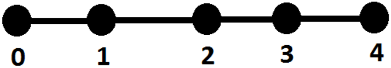

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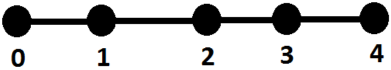

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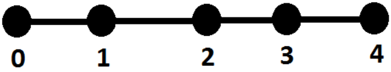

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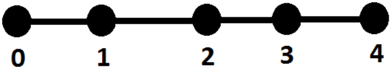

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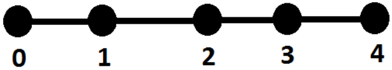

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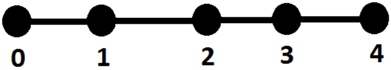

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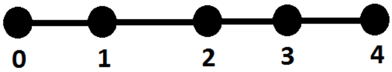

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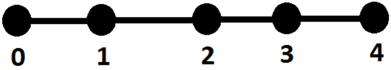

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&= \frac{4}{3}n - 2 \qquad (\text{versus } 2n - 3 \text{ for push})
\end{aligned}
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Known results

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- ✓ If Φ is Cheeger constant (conductance)
  and α is the vertex expansion (vertex isoperimetric number),
  Spread Time $\leq C \max\{\Phi^{-1} \log n, \alpha^{-1} \log^2 n\}$.

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Example: Chung-Lu, preferential attachment,.. .

Toward a more realistic model...

Asynchronous Rumor Spreading Protocols

Boyd, Ghosh, Prabhakar, Shah'06

1. A simple connected graph and each node has a Poisson clock of rate 1
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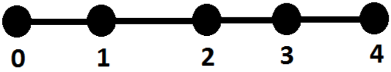

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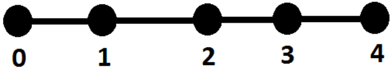

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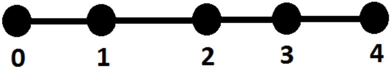

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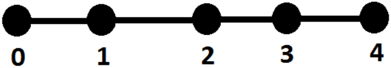

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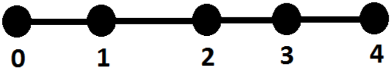
Example: Asynchronous push-pull on a path



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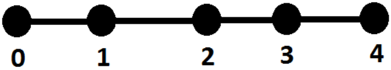

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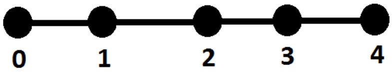

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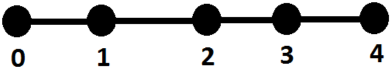

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- ✓ For every graph G
  push-pull: w.h.p.

$$ST_{asynch} = O(ST_{synch} + \log n)$$

$$\frac{ST_{synch}}{ST_{asynch}} \leq \sqrt{n} polylog(n)$$

Toward a "bit" more realistic model...

Multiple-Rate Asynchronous Rumor Spreading

P. and Ramezani'19

- ✓ Each node u has a Poisson clock of rate r_u chosen from a given distribution
- ✓ At the beginning, one vertex knows a rumor
- ✓ As soon as the Poisson clock of a vertex rings, it pushes (pulls) the rumor to (from) a random neighbor contacts a random neighbor.

Spread Time$ST(\varepsilon)$: For every $\varepsilon \in [0, 1)$, this is the first time when $(1 - \varepsilon)$fraction of nodes gets informed,

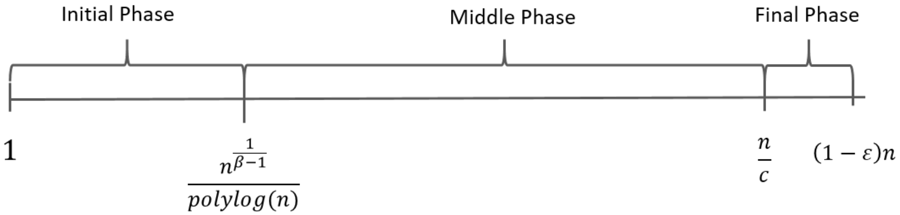
Our results

Theorem (P., Ramezani'19+)

Suppose that R is a power law prob. dist. with exponent $\beta \in (2,3)$. Let us consider the push-pull protocol on an n-node complete graph. Then, with constant probability, we have

$$ST(\varepsilon) = O(1 + \log(1/\varepsilon))$$

Proof Sketch



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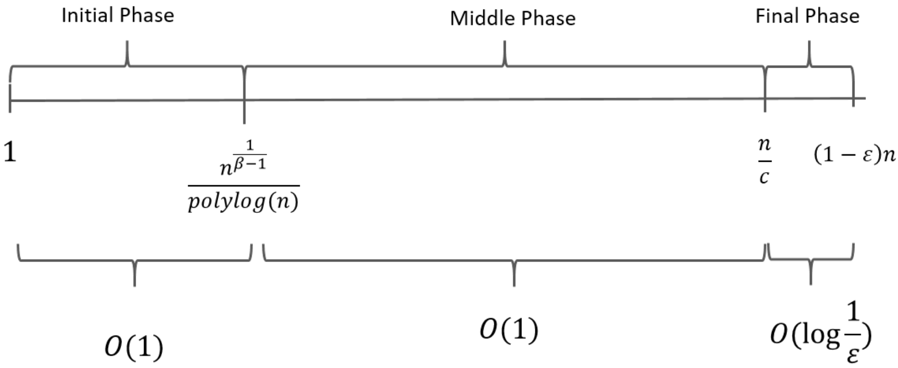

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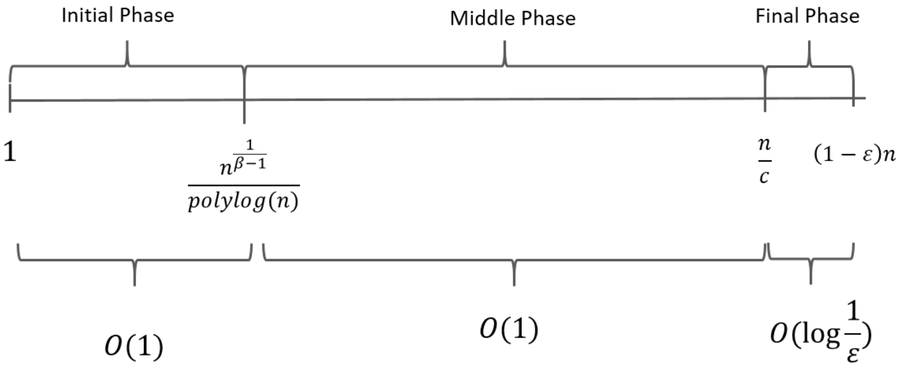

Expected total time: $O(1) + O(1) + O(\log 1/\varepsilon)$

Proof Sketch: Initial Phase

- Each node u was assigned a random number r_u from $R \propto k^{-\beta}$, $\beta \in (2, 3)$

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$$w_i := \begin{cases} \min\left\{\left(\frac{w_{i-1}}{2^{(i-1)/2}}\right)^{\frac{1}{\beta-2}}, \lfloor (n/\log^2 n)^{\frac{1}{\beta-1}} \rfloor\right\} & \text{if } i \text{ is odd,} \\ \frac{w_{i-1}}{2^{i/2}} & \text{otherwise.} \end{cases} \quad (1)$$

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- Corresponding to each w_i, define random variable

$$T_i := \begin{cases} \min\{t : |I_t| \geq w_i\} & \text{if } i \text{ is even,} \\ \min\{t : \exists u \in I_t \text{ with } r_u \geq w_i\} & \text{otherwise.} \end{cases}$$

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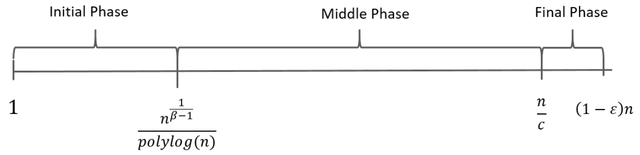

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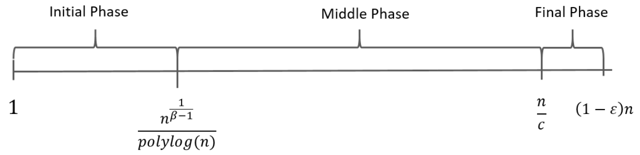


Remark

Random variable T_{2k} stochastically dominates the time is required for the initial phase.

Lemma

1. $\mathbb{E}[T_0] = O(1)$
2. *for some constant* C, $\mathbb{E}[T_{i+1} - T_i] \leq C^{-i}$

Corollary

$$\mathbb{E}[T_{2k}] = \mathbb{E}[T_{2k} - T_0] + \mathbb{E}[T_0] = \sum_{i=0}^{2k-1} \mathbb{E}[T_{i+1} - T_i] + \mathbb{E}[T_0]$$

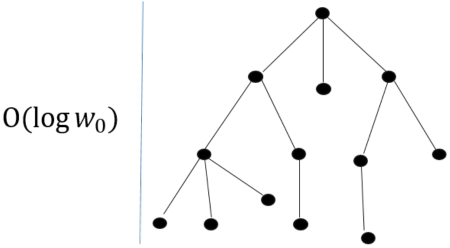
$$= \sum_{i=0}^{2k-1} \mathbb{E}[\mathbb{E}[T_{i+1} - T_i | T_i]] + \mathbb{E}[T_0] = O(1)$$

Lemma

1. $\mathbb{E}[T_0] = O(1)$
2. *for some constant* C, $\mathbb{E}[T_{i+1} - T_i] \leq C^{-i}$

Proof of 1.

$$T_0 = \min\{t : |I_t| \geq w_0 = 2^{\frac{4(\beta-2)}{(3-\beta)^2}+1}\}$$



$\Rightarrow \mathbb{E}[T_0] = O(1)$ □

Proof of 2.

Fix an even i, then

$$T_i = \min\{t : |I_t| \geq w_i\}$$
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Proof of 2.

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Since $|I_t| = o(n)$, for every $t \in [T_{i-1}, T_i]$,

$$\underbrace{\sum_{u \in I_t} \frac{r_u(n - |I_t|)}{n}}_{\text{Poission Rate(push attempt)}} \geq w_{i-1}(1 - o(1)) > \frac{w_{i-1}}{2}$$

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✓ Only consider the push protocol, to inform w_i new nodes

$$\mathbb{E}[T_{i+1} - T_i | T_i] \leq \frac{2w_i}{w_{i-1}} = 2^{-i/2}$$

$$\Rightarrow \mathbb{E}[T_{i+1} - T_i] = \mathbb{E}[\mathbb{E}[T_{i+1} - T_i | T_i]] \leq C^{-i}$$

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$\Rightarrow \mathbb{E}[T_{i+1} - T_i] = \mathbb{E}[\mathbb{E}[T_{i+1} - T_i | T_i]] \leq C^{-i}$
Similar technique works for odd i's. □

Our results

Theorem (P., Ramezani'19+)

Suppose that R is a prob. dist. with mean $\mu = O(1)$ and bounded variance. Let us consider the push protocol on an n-node complete graph. Then,

- ✓ $\mathbb{E}[ST(0)] = \frac{2\log n}{\mu} + \omega(1)$
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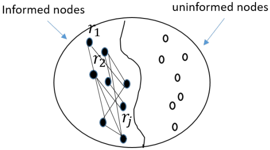

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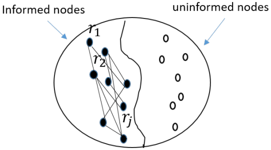

$S_j = r_1 + r_2 + \cdots + r_j$

t_j: required time to inform $(j+1)$-th node
t_j: exponentially distributed with rate $\frac{n-j}{n-1} S_j$

$$\mathbb{E}[t_j] = \mathbb{E}[\mathbb{E}[t_j | S_j]] = \mathbb{E}[\frac{n-1}{(n-j)S_j}] = \frac{n-1}{n-j} \underbrace{\mathbb{E}[\frac{1}{S_j}]}_{\text{it needs more cal.}} \sim \frac{n-1}{(n-j)\mu j}$$

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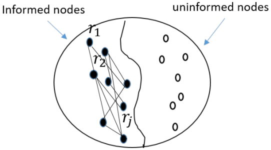

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$$\mathbb{E}[ST(0)] = \sum_{j=1}^{n-1} \mathbb{E}[t_j] = \frac{2\log n}{\mu} \pm O(1)$$

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Theorem (P., Ramezani'19+)

Suppose that R is a power law prob. dist. with exponent $\beta \in (2, 3)$. *Let us consider the push-pull protocol on an n-node complete graph. Then, with constant probability, we have*

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$$ST(0) = \frac{2\log n}{\mu} + \omega(\sqrt{\log n})$$

Multiple-Call Rumor Spreading (synch. version)

Panagiouto, P., Sauerwald'13

- ✓ Each node u was assigned a random number r_u chosen from a given distribution
- ✓ At the beginning, one vertex knows a rumor
- ✓ In each round, node u pushes (pulls) the rumor to (from) r_u random neighbors.

Spread Time: the first time when all nodes become informed.

Asynchronous vs Synchronous

algorithm	distribution	multiple-call	multiple-rate
push	$\mathbb{E}[R] = \mu < \infty, \mathbb{V}[R] < \infty$	$ST(0) = \frac{\log n}{\log(1+\mu)} + \frac{\log n}{\mu} \pm o(\log n)$	$ST(0) = \frac{\log n}{\mu} \pm o(\log n)$
push-pull	R is a power law with $\beta \in (2, 3)$	$ST(\varepsilon) = \Theta(\log\log n)$	$ST(\varepsilon) = \Theta(1)$

Asynchronous vs Synchronous

algorithm	distribution	multiple-call	multiple-rate
push	$\mathbb{E}[R] = \mu < \infty, \mathbb{V}[R] < \infty$	$ST(0) = \frac{\log n}{\log(1+\mu)} + \frac{\log n}{\mu} \pm o(\log n)$	$ST(0) = \frac{\log n}{\mu} \pm o(\log n)$
push-pull	R is a power law with $\beta \in (2, 3)$	$ST(\varepsilon) = \Theta(\log \log n)$	$ST(\varepsilon) = \Theta(1)$

The asynchronous model propagates the rumor much faster

Any Question?

Thank you!

alipourmiri@gmail.com