Some Old and New Open Questions in Discrete Isoperimetry Problem

Amir Daneshgar

Department of Mathematical Sciences Sharif University of Technology Tehran, Iran daneshgar@sharif.ir

Abstract

Isoperimetry is an ancient topic in mathematics whose roots can be traced back to Euclid (around 300 BC). Hence, any attempt to try to cover even a portion of what we know today in a one-hour talk would be fruitless!

This talk is mainly about an area that may be called *spectral discrete isoperimetry* and will be presented in three parts. The first part is a strive to introduce the basics of this fascinating subject, concentrating on examples and main ideas.

In the second part I will try to discuss a number of known results. Needless to say, I will be choosy and will try to focus on results that in my opinion can reveal different aspects of this multifaceted topic. These will cover theoretical findings as higher Cheeger inequalities, computational complexity issues and relations to metric embeddings and combinatorial optimization, as well as some relations to applied topics as clustering and image segmentation. My main objective in this part is to try to show that there is a deep interplay between theoretical and practical aspects of the subject leading to a better understanding and analysis of the problems.

In the last part I will try to pick a couple of open problems that can reveal how this subject can be praised and studied both from theoretical and practical aspects. These will definitely cover *mean-higher Cheeger inequalities* and *supergeometric graphs* that will be discussed within the time limitations.