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The n-Kronecker Modules

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The n-Kronecker modules are the representations of the n-Kronecker quiver; this is the quiver with two vertices, namely a sink and a source, and n arrows. The n-Kronecker modules have to be considered as basic objects in mathematics. As an introduction for the two lectures, we will discuss the relevance of the n-Kronecker modules in representation theory and outline essential features. We will recall that the case n = 2 has been studied a long time ago in various disguises: by Weierstrass and Kronecker, by Hilbert and Grothendieck, and by many other mathematicians; this is the prototype of a tame module category. But not much is known for n $\geq$ 3.

The main part of the first lecture will be devoted to the role of bristles: these are the indecomposable modules of length 2. As we will show there is an abundance of n-Kronecker modules which are generated by bristles.

In the second lecture we will determine the elementary 3-Kronecker modules. Let us recall that a regular representation of a quiver is said to be elementary provided it is non-zero and not a proper extension of regular representations. Of course any regular representation has a filtration whose factors are elementary. It turns out that the elementary 3-Kronecker modules are either tree modules or circle modules, thus determined by combinatorial invariants and at most one scalar.