Minimalist's Electromagnetism

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That the universal constancy of the speed of light is a consequence of Maxwell's equations is common knowledge.

We show that the converse is also true. That is, electromagnetism and electrodynamics in all their details can be derived from the simple assumption that the speed of light is a universal constant.

The consequences reach far.

Conventional EM and ED are observation based. The alternative we propose spares all observational foundations of EM and ED only to reintroduce them as theoretically derived and empiricism-free laws of Nature.

Simplicity is beauty and there are merits to it.

For instance, if $\nabla . B = 0$ emerges as a theoretical demand of the formalism, then nonexistence of magnetic monopoles will be a proven theorem and a reality.

Or, if Poisson's equation is derived from some first principles, then the inverse square law of Coulomb force becomes an exact law as long as the accepted first principles are tenable.

Electromagnetism (EM) and electrodynamics are observation based. They are founded on:

- Coulomb's law, 1784
- Ampere's law, 1826
- Faraday's law, 1831
- Displacement current of Maxwell to conform with charge-current conservation, 1865
- The fact that all magnet Found in Nature are all dipoles, time immemorial
- And Lorentz force equation.

$$\boldsymbol{\nabla}.\boldsymbol{B} = \boldsymbol{0}, \qquad \boldsymbol{\nabla}\times\boldsymbol{E} + \frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} = 0,$$

$$\boldsymbol{\nabla}.\boldsymbol{E} = 4\pi\rho, \qquad \boldsymbol{\nabla}\times\boldsymbol{B} - \frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t} = 4\pi\boldsymbol{J}.$$

$$\frac{\partial\rho}{\partial t} + \nabla.J = 0$$

Lorentz $\boldsymbol{F} = e\,[\,\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\,]$

Manifest Lorentz covariance:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

$$F_{\mu\sigma,\rho} + F_{\sigma\rho,\mu} + F_{\rho\sigma,\mu} = 0,$$

$$F^{\mu\sigma},\sigma = 4\pi J^{\mu}, \quad J^{\mu}{}_{,\mu} = 0,$$

$$\text{Lorentz } F^{\mu} = e\, F^{\mu\nu} U_{\nu}.$$

Observation based beginnings, however, have an Achilles' heel. What if there are escapees from observations and if detected change one's understanding of the Nature.

The question of magnetic monopoles is one such case. All magnets found in Nature are dipoles. Therefore, one has concluded that the magnetic field is divergence free.

But what if there is one magnetic monopole somewhere in the Universe, and what if such supposition is expounded and theorized by persons of the reputation of Dirac.

We replace all conventional foundations of EM with much simpler ones, though themselves observation based.

Our first principles are:

1. ***There is light and its speed is a universal constant,***
2. ***There are particles prone to acceleration.***

The first principle implies Lorentz invariance of spacetime intervals

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

To implement the second principle, assume a test particle with 4-mom

$$p^\mu = mc\frac{dx^\mu}{d\tau}, \quad p^\mu p_\mu = (mc)^2$$

Calculate acceleration and perform the following algebra

$$\frac{dp^\mu}{d\tau} = \frac{\partial p^\mu}{\partial x^\nu}\frac{dx^\nu}{d\tau} = \frac{1}{mc}\frac{\partial p^\mu}{\partial x^\nu}\, p^\nu.$$

Define $\quad eF_{\mu\nu} = \frac{\partial p_\mu}{\partial x^\nu}, \quad e$ const.

$$\frac{d}{d\tau}\left(p^\mu p_\mu\right) = e\, F_{\mu\nu} p^\mu p^\nu = 0,$$

$$F_{\mu\nu} = -F_{\nu\mu}, \text{ antisymmetric.}$$

From the definition and antisymmetry of $F_{\mu\nu}$:

$$\frac{\partial^2 F_{\mu\nu}}{\partial x^\sigma \partial x^\nu}$$

Antisymmetry of *F* leads to

$$F_{\mu\nu,\rho} + F_{\nu\rho,\mu} + F_{\rho\mu,\nu} = 0. \qquad (1)$$

For later use take 4- divergence of $F^{\mu\nu}$ and set it equal to some vector J^{μ} :

$$F^{\mu\nu}{}_{,\nu} = 4\pi J^{\mu}, \quad \text{with} \quad J^{\mu}{}_{,\mu} = 0. \qquad (2)$$

Separate the time and space components of equation of motion

$$\frac{dp^0}{d\tau} = F^{0i}U_i \qquad (3)$$

$$\frac{dp^i}{d\tau} = F^{i0}p_0 + F^{ij}p_j. \qquad (4)$$

Denote $F^{0i} = -E_i$ and $F^{ij} = \varepsilon^{ijk}B_k$. Display *F*

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

Part of the job is done. It suffices to interpret ***E, B****, and F* as the electric vector, the magnetic vector, and the field tensor that one is familiar with in Maxwell's equations. One will find Eq (1) as the homogenous pair of Maxwell's equations. And Eqs (3) and (4) as the evolution eqs of energy and 3-momentum of the particle.

Up to this point we have dealt with kinematics . To introduce dynamics we argue that the field *F* act on charged particles. There should be a reaction from particles on field. We can construct two divergence free 4-vectors from the field and from the particles, namely:

$$F^{\mu\nu}{}_{,\nu} \quad \text{and} \qquad J^{\mu} = \sum_a e_a dx^{\mu}/dt.$$

Our third assumption is to equate these two vectors

$$F^{\mu\nu}{}_{,\nu} = 4\pi J^{\mu}\,, \quad J^{\mu}{}_{,\mu} = 0. \qquad (2)$$

This is the non-homogenous pair of maxwell's eqs and contains dynamics of EM field.

Conclusion

We have began with three observation based but simple principles.

1. Speed of light is a universal constant,
2. There are particles prone to acceleration,
3. Particles interact with each other through a field.

We find the spacetime should necessarily be pervaded by the unique rank 2 tensor EM field.

The force on a test particle is necessarily Lorentz force and unique.

All observation based laws of conventionally formulated EM emerge as proven theorems.

In particular the theoretically derived $\nabla . B = 0$ implies non-existence of magnetic monopoles

And the theoretically derived $\nabla . E = \rho$ implies that the inverse square force of coulomb is exact.

The following is a quotation from Hobson, General Relativity:

The fact that the Einstein equations predict the equation of motion is remarkable and should be contrasted with the situation in electrodynamics. In the latter case, the Maxwell equations for the electromagnetic field do not contain the corresponding equation of motion for a charged particle, which has to be postulated separately. The origin of this distinction between gravity and electromagnetism lies in the non-linear nature of the Einstein equations. The physical reason for this non-linearity is that the gravitational field itself carries energy–momentum and can therefore act as its own source, …

Here, however, we conclude that both Maxwell's equations and the equation of motion of a charged particle both follow from the same set of principles and therefore from each other.

It is thought provoking how three simple propositions, constancy of c, existence of particles prone to acceleration, and their mutual interactions can lead to the complex and multi component structure EM and ED.

Here are some questions:

- Why there is no provision for scalar or vector fields in the formalism?
- In case there are scalar and vector fields in Nature, and one wishes to inspect them by a path similar to what is done here, should one look for another universal constant, a different kind of test particle, or both?
- Are other universal constants of Nature as resourceful and as mysterious as *c*?
- Do other forces of Nature admit explanations of the sort adopted here for EM.