On uniserial dimension and ascending chains of submodules

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We define uniserial dimension of modules which is an ordinal valued number dimension to measure how far a module is far from being uniserial. According to our published paper in Journal Algebra, for every ordinal α, there exists a module of uniserial dimension α. There, we see existing of uniserial dimension for a module has very close relation to ascending chains of its submodules. In fact, it is shown a ring R is semisimple artinian if and only if every right module has uniserial dimension if and only if for every right module M and ascending chain $M_1 \leq M_2 \leq \cdots$ of submodules of M there exists $n \geq 1$ such that $M/M_n \cong M/M_i$, for all $i \geq n$. Also, according to another published paper, a commutative ring is Noetherian if and only if every finitely generated module has uniserial dimension if and only if for every finitely generated module M and ascending chain $M_1 \leq M_2 \leq \cdots$ of submodules of M there exists $n \geq 1$ such that $M/M_n \cong M/M_i$, for all $i \geq n$. We are going to discuss on general case and try to characterize rings whose finitely generated modules have uniserial dimension. (This is a joint work with A. Ghorbani).