Meshless Methods for Solving Operator Equations

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Abstract: Most numerical methods for solving PDE problems or general operator equations arrive at linear systems of equations. Each such equation can be viewed as a *test* of a *trial function*, and different classes of methods have different spaces of trial functions and different test strategies. Enlarging the space of trial functions should increase the achievable accuracy, and increasing the number of test equations should stabilize the calculation. These two ingredients, *consistency* and *stability*, are the basic building blocks of any analysis of numerical methods for solving operator equations.

The talk defines these notions in full generality and then formalizes *well-posedness* of problems in weak or strong form and focuses on well-posed linear operator equations, including boundary value problems for elliptic partial differential equations. To stay general, all sorts of trial spaces are allowed, e.g. finite elements, polynomials, eigenfunctions, and translates of kernels. The central result will be that all well-posed problems have uniformly stable discretizations: for each chosen trial space there exists a selection of test equations such that the stability of the final system is dependent only on the well-posedness of the given problem. Furthermore, the convergence rates are then determined completely by consistency. In particular, they can be reduced to rates of approximation of derivatives of the solution by derivatives of trial functions. This yields new and simple proofs for a variety of existing techniques for PDE solving, including Kansa's unsymmetric collocation, Generalized Finite Elements and Atluri's Meshless Local Petrov Galerkin Method. It is also shown that for a fixed trial space, weak formulations have a slightly better convergence rate than strong formulations, but at the expense of numerical integration.

Unfortunately, there are important methods that do not fall under the theory presented so far, namely generalized Finite Differences and the Direct Meshless Local Petrov Galerkin technique. If time permits, it will be shown how to treat these cases by evaluating consistency and stability numerically, thus bypassing the missing theory. But there will surely be not enough time to deal with time-dependent problems.

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