Symmetric graphs, finite groups and block designs

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An *s-arc* of a graph is a sequence of $s+1$ vertices such that any two consecutive vertices in the sequence are adjacent and any three consecutive vertices are distinct, where $s \geq 0$ is an integer. A graph Γ is called (G, s)*-arc-transitive* if G is a subgroup of the automorphism group of Γ that is transitive on the set of s-arcs of Γ. A 1-arc is usually called an *arc*, and a $(G, 1)$-arc-transitive graph is called a *G-symmetric graph*.

Beginning with Tutte's seminal work (1947) on cubic symmetric graphs, the study of symmetric graphs and highly arc-transitive graphs has a long history but is still an active research area. The theory of finite groups plays an important role in studying various problems in this area. In the case when the group G is imprimitive on the vertex set of Γ, transitive block designs are also involved by a 'geometric' approach introduced by A. Gardiner and C. E. Praeger.

I will review some recent and not-so-recent results on symmetric graphs with a focus on the imprimitive case. The talk consists of three parts:

1. Symmetric graphs

This part will be an introduction to symmetric graphs and highly arc-transitive graphs. Connections between imprimitive symmetric graphs and transitive block designs will also be discussed.

2. Unitary graphs and affine graphs

In this part I will talk about two families of symmetric graphs such that the block designs involved are 2-point-transitive linear spaces.

3. Classification of a family of symmetric graphs

In the last part I will present a recent classification of a family of imprimitive symmetric graphs with complete quotients.