

Homological Aspects of Categorical Algebras (Special Bound Quiver Algebras)

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Let R be a ring and \mathcal{Q} be a finite quiver. We gave an explicit formula for the injective envelopes and projective precovers in the category $\text{Rep}(\mathcal{Q}_I, R)$ of bound representations of \mathcal{Q} by left R -modules, where I is a combination of monomial and commutativity relations. Some applications would be provided. In particular, it was shown that if R is an Iwanaga-Gorenstein ring, then so are these bound quiver algebras.

We also extended our formula to all terms of the minimal injective resolution of $R\mathcal{Q}$. Using such descriptions, we studied the Auslander-Gorenstein property of path algebras. In particular, we proved that the path algebra $R\mathcal{Q}$ is k -Gorenstein if and only if $\mathcal{Q} = \overrightarrow{A}_n$ and R is a k -Gorenstein ring, where n is the number of vertices of \mathcal{Q} .

In fact we studied some homological aspects of path algebras. Also we studied some homological Aspects of $U_N(R)$.

Note that the classical representation theory of quivers considers finite quivers and assume that the base ring is algebraically closed field and that all vector spaces involve are finite dimensional. But we wrote our results in greater generality (not just fields) for some reasons: One of the advantage of working in the category of representations, with value in the category of R -modules when R is an arbitrary ring (not only field) is to study the category of representations of a quiver \mathcal{Q} with relations over a field. On the other hand by working with them one can reproof and extend some interesting results in the literature.