

Logic and Arithmetic,

In this lecture we talk about how some general arithmetical interests can lead to mathematical logical considerations about the natural numbers. Giuseppe Peano, David Hilbert, Kurt Godel and Thoralf Skolem's works in the early 20th century, are important examples of this type of thinking. Incompleteness of the Peano axioms to derive all true statements of the natural numbers (Godel's first incompleteness theorem) and more generally, non Axiomatizability of \mathbb{N} (an implication of Godel's second incompleteness theorem), non algorithmic solvability of all Diophantine equations (MRDP's theorem) and insufficiency of first order languages to characterize \mathbb{N} uniquely (Skolem), are instances of theorems with great effects on the later developments of mathematical logic in 20th century which have their roots in purely arithmetical viewpoints. Godel, Church and Turing's works made it possible to formulate exact notions of algorithm and prove some important theorems about it which in the hands of Davis, Putnam, Robinson and Matijasevic in the later decades, led to a complete solution of the undecidability of solvability of Diophantine equations, what is called Hilbert's tenth problem (1970). If the time permits we will talk about later activities and new topics that are continuations of these lines of thought, for example Paris and Harrington's independence result for Peano's axioms (1977), Prikh's discovery of the undefinability of totality of exponentiation in bounded arithmetic (1971) and Shepherdson's independence results for weak arithmetic (1964).