

# Programme in Noncommutative Geometry at IPM

Masoud Khalkhali

## Synopsis

Noncommutative Geometry is a rapidly growing interdisciplinary field of modern mathematics. The subject, originally conceived some 30 years ago by Alain Connes, is now turning into a major enterprise in modern mathematics and its applications to theoretical physics. Starting from its original roots in operator algebras, index theory, and quantum mechanics, the subject has already made major inroads into areas of modern mathematics such as algebraic topology (the Novikov and Baum-Connes conjectures), index theory of elliptic operators (extensions of the index theorem of Atiyah and Singer to highly singular spaces), geometric group theory, algebra, and number theory. Its applications to theoretical physics include solid state physics, the Standard Model of elementary particles, renormalization in quantum field theory, string theory, and gauge theory.

Noncommutative geometry, being a very young subject with many connections to diverse fields, has much potential for growth and offers a unique opportunity to young researchers to engage in state of the art research after an intensive training in a relatively short span of time.

The *Noncommutative Geometry Programme* at IPM will guide the participants to the frontiers of research in noncommutative geometry. We shall offer courses and training in noncommutative geometry in combination with a variety of background material from relevant areas in mathematics. The program will last about nine months and is divided into two semesters. The programme starts in September 2010 and will run till July 2011.

## The Programme

- *First Semester*

The main activity in the first semester consists of two courses, a research seminar (journal club), minicourses, and daily tutorials as follows:

*Basic noncommutative geometry:* This course will introduce the rudiments of the subject with an emphasis on understanding the topological aspects of the subject. Topics include: the two paradigms of noncommutative spaces:  $C^*$ -algebras and von Neumann algebras, Gelfand-Naimark and Serre-Swan theorems, noncommutative quotients, groupoids, and Morita equivalence, cyclic cohomology,  $K$ -theory and  $K$ -homology, Cones-Chern character maps and Connes' index formula, applications. Marking will be based on assignments, a final exam, and a term paper with presentation.

*$K$ -theory and index theory:* This can be a joint course with the algebraic geometry program. Topological  $K$ -theory and the Bott periodicity theorem, characteristic classes and Chern-Weil theory, the index problem, pseudodifferential operators, the Atiyah-Singer index theorem, examples: Gauss-Bonnet, Hirzebruch signature, and Riemann-Roch theorems.

*NCG journal club:* Students will present materials from selected original papers. We shall also invite local researchers in the field and nearby areas to present their results.

*Tutorials:* We expect to run daily one hour tutorials for the duration of the master class. During these sessions students will have a chance, and will be encouraged, to further discuss and debate the topics among themselves and with instructors. These sessions will be usually supervised by instructors, postdocs, or senior PhD students around.

*Minicourses:* We shall also organize short intensive training sessions to introduce various background material as they might be needed. This can include: basics of differential geometry, algebraic topology, homological algebra, operator algebras; as well as topics in theoretical physics: classical and quantum mechanics, gauge theory, quantum field theory, the standard model.

- *Second Semester*

The second semester will be focused on metric aspects of noncommutative geometry and the impact of spectral geometry on the subject. We shall have a topics course on spectral geometry as well as a course on spectral triples and the local index formula. The rest of the activity will be as in the first semester (minicourses, seminar, and tutorials).

*Topics in spectral geometry:* Laplacian on Riemannian manifolds, Weyl's law

on the asymptotic distribution of eigenvalues of the Laplacian, heat equation techniques in index theory, spectral zeta functions.

*Topics in spectral triples:* Analytic background (the Dixmier trace and non-commutative residue), examples of spectral triples, local index formula, real structures, Connes' axioms for noncommutative manifolds and the reconstruction theorem, the spectral action principle.

*Seminar on spectral triples:* Students will present selected materials from original papers.

## **Organization**

The program will be supervised by Professor Masoud Khalkhali with the collaboration of Dr. Eaman Eftekhari on behalf of IPM.

## **Certificate**

Applicants who have successfully completed the full program will receive a certificate from IPM. Moreover, upon the permission of participant's university, course credits may be transferred to participants' host universities. Participants in the program will receive a stipend from the IPM for the period of the program.

## **Prerequisites**

Applicants are expected to have a basic understanding, at the first year graduate level, of analysis, geometry, topology, and algebra, and should be registered at the second year master level in their local universities. On an exceptional basis, applicants with a different background will be considered as well.

## **How to apply**

Applicants should send their transcripts and at least two letters of reference and cv to IPM. The deadline for application is 31 Tir 1389 (July 22, 2010).