

Combinatorial Optimization: a Bridge between Combinatorics and Algebra

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Main Theme

Correspondence of **Hypergraphs** in Combinatorics and
Squarefree Monomial Ideals in Algebra

Tool: Combinatorial Optimization

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Correspondence of **Hypergraphs** in Combinatorics and **Squarefree Monomial Ideals** in Algebra

Tool: Combinatorial Optimization

This bridge is too big. I am a passenger passing the bridge a few times and could see only a glimpse of its magnificence.

Hypergraphs

A **hypergraph** Γ is a collection of subsets of a set V with no inclusions among them. One may view the elements of V as **vertices** and the subsets of Γ as **edges** of the hypergraph.

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Hypergraphs play an important role in Combinatorics and in dealing with real discrete problems.

Example: Social networks.

Matrix presentation

Let $V = \{1, \dots, n\}$ and $\Gamma = \{F_1, \dots, F_m\}$.

We may identify F_i with the **incidence vector** $\mathbf{a}_i = (\alpha_{i1}, \dots, \alpha_{in})$ (column vector), where

$$\alpha_{ij} = \begin{cases} 1 & \text{if } j \in F_i, \\ 0 & \text{if } j \notin F_i. \end{cases}$$

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$$\alpha_{ij} = \begin{cases} 1 & \text{if } j \in F_i, \\ 0 & \text{if } j \notin F_i. \end{cases}$$

Then Γ is uniquely determined by the **incidence matrix**

$$\mathbf{M} = (\mathbf{a}_1, \dots, \mathbf{a}_m).$$

This matrix presentation allows us to use tool of Combinatorial Optimization to study hypergraphs.

Cover

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Let \mathbf{b} be the incidence vector of G and $\mathbf{1}_n := (1, \dots, 1) \in \mathbb{N}^n$. Then

$$|G| = \mathbf{1}_n \cdot \mathbf{b}.$$

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Proposition: $\tau(\Gamma) = \min\{\mathbf{1}_n \cdot \mathbf{b} \mid \mathbf{b} \in \mathbb{N}^n, \mathbf{M}^T \cdot \mathbf{b} \geq \mathbf{1}_m\}.$

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Let \mathbf{c} be the incidence vector of the index set $\{i \mid F_i \in S\}$. Then

$$|S| = \mathbf{1}_m \cdot \mathbf{c},$$

The edges of S are disjoint iff $\sum_{F_i \in S} \mathbf{a}_i \leq \mathbf{1}_n$ iff $\mathbf{M} \cdot \mathbf{c} \leq \mathbf{1}_n$.

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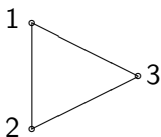
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Proposition: $\nu(\Gamma) = \max\{\mathbf{1}_m \cdot \mathbf{c} \mid \mathbf{c} \in \mathbb{N}^m, \mathbf{M} \cdot \mathbf{c} \leq \mathbf{1}_n\}$.

Example

Let $\Gamma = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$:



Γ has three minimal covers $\{1, 2\}, \{1, 3\}, \{2, 3\}$, hence $\tau(\Gamma) = 2$.

Γ has three maximal matchings of one edge $\{1, 2\}, \{1, 3\}, \{2, 3\}$, hence $\nu(\Gamma) = 1$.

König property

Every cover meets the edge of a matching at different vertices:

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Another point of view:

$$\begin{aligned} \nu(\Gamma) &\leq \max\{\mathbf{1}_m \cdot \mathbf{c} \mid \mathbf{c} \in \mathbb{R}_+^m, \mathbf{M} \cdot \mathbf{c} \leq \mathbf{1}_n\} \\ &= \min\{\mathbf{1}_n \cdot \mathbf{b} \mid \mathbf{b} \in \mathbb{R}_+^n, \mathbf{M}^T \cdot \mathbf{b} \geq \mathbf{1}_m\} \leq \tau(\Gamma). \end{aligned}$$

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If $\nu(\Gamma) = \tau(\Gamma)$, one says that Γ has the **König property**.

König: Bipartite graphs have this property.

Expanding Hypergraphs

To construct new hypergraphs one can expand a vertex v to k vertices as follows:

1. Replacing v by k new vertices v_1, \dots, v_k ,
2. Replacing every edge F containing v by k new edges $(F \setminus v) \cup v_1, \dots, (F \setminus v) \cup v_k$.

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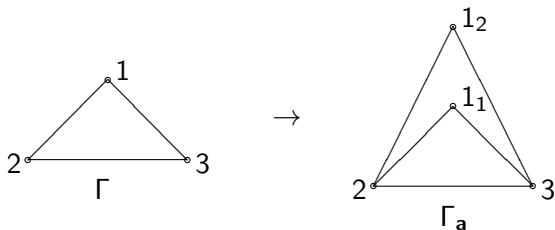
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For $\mathbf{a} = (\alpha_1, \dots, \alpha_n) \in \mathbb{N}^n$, we define $\Gamma_{\mathbf{a}}$ as the hypergraph obtained from Γ by expanding every vertex i to α_i vertices, $i = 1, \dots, n$.

Example

Let $\Gamma = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ and $\mathbf{a} = (2, 1, 1)$.

Then Γ_a is obtained by expanding the vertex 1 to two vertices $1_1, 1_2$:



Fractional covering and matching

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Lemma. For all $\mathbf{a} \in \mathbb{N}^m$ we have

$$\begin{aligned}\nu(\mathbf{a}) &= \max\{\mathbf{1}_m \cdot \mathbf{c} \mid \mathbf{c} \in \mathbb{N}^m, \mathbf{M} \cdot \mathbf{c} \leq \mathbf{a}\}, \\ \tau(\mathbf{a}) &= \min\{\mathbf{a} \cdot \mathbf{b} \mid \mathbf{b} \in \mathbb{N}^n, \mathbf{M}^T \cdot \mathbf{b} \geq \mathbf{1}_m\}.\end{aligned}$$

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The fractional covering and matching numbers is defined by

$$\begin{aligned}\nu^*(\mathbf{a}) &:= \max\{\mathbf{1}_m \cdot \mathbf{c} \mid \mathbf{c} \in \mathbb{R}_+^m, \mathbf{M} \cdot \mathbf{c} \leq \mathbf{a}\}, \\ \tau^*(\mathbf{a}) &:= \min\{\mathbf{a} \cdot \mathbf{b} \mid \mathbf{b} \in \mathbb{R}_+^n, \mathbf{M}^T \cdot \mathbf{b} \geq \mathbf{1}_m\}.\end{aligned}$$

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Proposition: $\nu(\mathbf{a}) \leq \nu^*(\mathbf{a}) = \tau^*(\mathbf{a}) \leq \tau(\mathbf{a})$.

Squarefree monomial ideals

Let $K[X] = K[x_1, \dots, x_n]$ be a polynomial ring over a field K .

For $\mathbf{a} = (\alpha_1, \dots, \alpha_n) \in \mathbb{N}^n$, set $x^{\mathbf{a}} := x_1^{\alpha_1} \cdots x_n^{\alpha_n}$.

We call $x^{\mathbf{a}}$ **squarefree** if $x^{\mathbf{a}}$ is not divided by any square. In this case, $\mathbf{a} \in \{0, 1\}^n$, and we may associate with \mathbf{a} the set $F = \{i \mid \alpha_i = 1\}$.

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Let $I = (x^{\mathbf{a}_1}, \dots, x^{\mathbf{a}_m})$ be a squarefree monomials, i.e. $x^{\mathbf{a}_1}, \dots, x^{\mathbf{a}_m}$ are squarefree. Let F_1, \dots, F_m be the sets associated with $\mathbf{a}_1, \dots, \mathbf{a}_m$. Then I is determined by the hypergraph $\Gamma = \{F_1, \dots, F_m\}$.

We call I the **edge ideal** of Γ .

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Let $I = (x^{\mathbf{a}_1}, \dots, x^{\mathbf{a}_m})$ be a squarefree monomial ideal, i.e. $x^{\mathbf{a}_1}, \dots, x^{\mathbf{a}_m}$ are squarefree. Let F_1, \dots, F_m be the sets associated with $\mathbf{a}_1, \dots, \mathbf{a}_m$. Then I is determined by the hypergraph $\Gamma = \{F_1, \dots, F_m\}$.

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This gives a correspondence between squarefree ideals and hypergraphs.

Example

Let $I = (x_1x_2, x_1x_3, x_2x_3) = (x^{\mathbf{a}_1}, x^{\mathbf{a}_2}, x^{\mathbf{a}_3})$,
where $\mathbf{a}_1 = (1, 1, 0)$, $\mathbf{a}_2 = (1, 0, 1)$, $\mathbf{a}_3 = (0, 1, 1)$.

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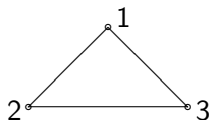
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Then $F_1 = \{1, 2\}$, $F_2 = \{1, 3\}$, $F_3 = \{2, 3\}$.

Hence I is the edge ideal of the graph $\Gamma = \{F_1, F_2, F_3\}$:



Symbolic powers

Let I be the edge ideal of a hypergraph Γ .

Let C_1, \dots, C_s be the minimal covers of Γ .

Then I has the decomposition

$$I = P_1 \cap \dots \cap P_s$$

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The k -th **symbolic power** of I is the ideal

$$I^{(k)} := P_1^k \cap \dots \cap P_s^k.$$

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We always have $I^k \subseteq I^{(k)}$.

Mengerian hypergraph

Problem: When $I^k = I^{(k)}$ for all $k \geq 1$?

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There are the following membership criteria:

Lemma: $x^{\mathbf{a}} \in I^k$ iff $\nu(\mathbf{a}) \geq k$.

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One calls Γ **Mengerian** if $\nu(\mathbf{a}) = \tau(\mathbf{a}) \forall \mathbf{a} \in \mathbb{N}^n$.

Herzog-Hibi-Tr-Zheng:

$I^k = I^{(k)}$ for all $k \geq 1$ iff Γ is a Mengerian hypergraph.

Integral closures

Let I be an ideal in a ring R . The **integral closure** of I is defined as the ideal

$$\bar{I} := \{f \in R \mid \exists f^d + g_1 y^{d-1} + \cdots + g_d = 0, g_j \in I^j\}.$$

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This notion has its origin also in Algebraic Geometry.

For a squarefree monomial ideal I , we have $I^k \subseteq \bar{I}^k \subseteq I^{(k)}$.

Problem: When do we have equality in the above inequalities?

Fulkersonian hypergraph

Recall that $\nu(\mathbf{a}) \leq \nu^*(\mathbf{a}) = \tau^*(\mathbf{a}) \leq \tau(\mathbf{a})$.

Lemma. $x^{\mathbf{a}} \in \overline{I^k}$ iff $\tau^*(\mathbf{a}) \geq k$.

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Tr: $\overline{I^k} = I^{(k)}$ for all $k \geq 1$ iff Γ is Fulkersonian.

One may expect that

$I^k = \overline{I^k}$ for all $k \geq 1$ iff $\nu(\mathbf{a}) = \tau^*(\mathbf{a}) \forall \mathbf{a} \in \mathbb{N}^n$?

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$I^k = \overline{I^k}$ for all $k \geq 1$ iff $\nu(\mathbf{a}) = \tau^*(\mathbf{a}) \forall \mathbf{a} \in \mathbb{N}^n$?

This is not true. So what is the condition for $I^k = \overline{I^k}$.

Integer round-down property

Let $\lfloor \nu^*(\mathbf{a}) \rfloor$ denote the integer round-down of $\nu^*(\mathbf{a})$. Then

$$\nu(\mathbf{a}) \leq \lfloor \nu^*(\mathbf{a}) \rfloor \leq \nu^*(\mathbf{a}).$$

Lemma: $x^{\mathbf{a}} \in \overline{I^k}$ iff $\tau^*(\mathbf{a}) \geq k$ iff $\lfloor \nu^*(\mathbf{a}) \rfloor \geq k$.

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We say that Γ has the **integer round-down property** if $\nu(\mathbf{a}) = \lfloor \nu^*(\mathbf{a}) \rfloor$ for all $\mathbf{a} \in \mathbb{N}^n$.

Tr: $I^k = \overline{I^k}$ for all $k \geq 1$ iff Γ has the integer round-down property.

Applications

Combinatorics: The above classes of hypergraphs were studied already in the 70' by Berge, Fulkerson, Lovasz, Schrijver, Seymour, Trotter, etc.

Algebra: The corresponding properties of monomial ideals have been studied only since the 90'.

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Consequences of the relationship:

1. Several new results on monomial ideals can be recovered by earlier results on hypergraphs.
2. New classes of monomial ideals or hypergraphs can be discovered by means of combinatorics or algebra, respectively.