Integral symbolic and adic topologies

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Let R be a commutative Noetherian ring and I an ideal of R. For a natural number n, the nth symbolic power $I^{(n)}$ (resp. the nth integral symbolic power $I^{\langle n\rangle}$) of I, is defined to be the union of $(I^n :_R s)$ (resp. the union of $(\overline{I^n} :_R s)$), where s runs in the multiplicatively closed subset $\bigcap_{\mathfrak{p}\in \mathrm{Min}(I)}(R\setminus \mathfrak{p})$. Also, the set of quintasymptotic primes of I, denoted by $\bar{Q}^*(I)$, is defined as

$\bar{Q}^*(I) = \{\mathfrak{p} \in \operatorname{Spec} R \mid$ there is a minimal prime z in $\hat{R}_{\mathfrak{p}}$ with $\mathfrak{p}\hat{R}_{\mathfrak{p}}$ minimal over $I\hat{R}_{\mathfrak{p}} + z\}$.

In this talk, we show that the topologies defined by the integral filtration $\{\overline{I^m}\}_{m\geq 1}$ and the symbolic integral filtration $\{I^{\langle m\rangle}\}_{m\geq 1}$ are equivalent, whenever $\bar{Q}^*(I)$ consists all of the minimal primes of I.

As an application of this result, by using Lipman-Sathaye's theorem we deduce that the symbolic integral topology $\{I^{\langle m\rangle}\}_{m\geq 1}$ is equivalent to the I-adic topology, whenever R is a regular ring. Moreover, a generalization of a result of Hartshorne, as well as a result of Zariski is given.

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