Strongly irreducible submodules in the Arithmetical and Noetherian modules

Reza Naghipour

Let R be a commutative ring and let M be an arbitrary R-module. In this talk, we will introduce the concept of strongly irreducible submodules of M and we will prove some properties of them, whenever M is either an arithmetical or a Noetherian module. In case when R is Noetherian and M is finitely generated, several characterizations of strongly irreducible submodules are included. Among other things, it is shown that when N is a submodule of M such that $N :_R M$ is not a prime ideal, then N is strongly irreducible if and only if there exist a submodule L of M and a prime ideal $\mathfrak{p}$ of R such that N is $\mathfrak{p}$-primary, $N \subsetneqq L \subseteq \mathfrak{p}M$ and for all submodules K of M either $K \subseteq N$ or $L_\mathfrak{p} \subseteq K_\mathfrak{p}$. In addition, we show that a submodule N of M is strongly irreducible if and only if N is primary, $M_\mathfrak{p}$ is arithmetical and $N = (\mathfrak{p}M)^{(n)}$ for some integer $n > 1$, where $\mathfrak{p} = \mathrm{Rad}(N :_R M)$ with $\mathfrak{p} \notin \mathrm{Ass}_R R/\mathrm{Ann}_R(M)$ and $\mathfrak{p}M \nsubseteq N$. As a consequence we deduce that if R is integral domain and M is torsion-free, then there exists a strongly irreducible submodule N of M such that $N :_R M$ is not a prime ideal if and only if there is a prime ideal $\mathfrak{p}$ of R with $\mathfrak{p}M \nsubseteq N$ and $M_\mathfrak{p}$ is an arithmetical $R_\mathfrak{p}$-module.
This talk is based on a joint work with M. Sedghi (preprint 2020).

University of Tabriz
Tabriz
Iran
E-mail address: naghipour@ipm.ir