

Abstract

In [8], Avramov and Buchweitz proved that for finitely generated modules M and N over a complete intersection local ring R , $\text{Ext}_R^i(M, N) = 0$ for all $i \gg 0$ implies $\text{Ext}_R^i(N, M) = 0$ for all $i \gg 0$. In the first chapter, we give some generalizations of this result. Indeed we prove the above mentioned result when (1) M is finitely generated and N is arbitrary, (2) M is arbitrary and N has finite length and (3) M is complete and N is finitely generated.

In the second chapter, we investigate the finiteness of Ext-indices for certain ring extensions. In this direction, we introduce some conjectures and discuss the relationships among them. We also prove these conjectures in some special cases. Furthermore, we prove that the trivial extension of an Artinian local ring by its residue class field is always of finite Ext-index, and prove a generalization of the Auslander-Reiten conjecture for this type of ring.

In the third chapter, we prove some other results about vanishing of Ext. Let M be a finitely generated module over a local ring R of characteristic $p > 0$. If $\text{depth}(R) = s$, then the property that M has finite projective dimension can be characterized by the vanishing of the functor $\text{Ext}_R^i(M, f^n R)$ for $s + 1$ consecutive values $i > 0$ and for infinitely many n . In addition, if R is a d -dimensional complete intersection ring, then M has finite projective dimension can be characterized by the vanishing of the functor $\text{Ext}_R^i(M, f^n R)$ for some $i \geq d$ and some $n > 0$.

Keywords

1. Regular ring
2. Complete intersection ring
3. Gorenstein ring
4. Cohen-Macaulay ring
5. AB ring
6. Trivial extension
7. Auslander-Reiten conjecture
8. Frobenius functor
9. Projective dimension
10. Injective dimension
11. Flat dimension