Abstract

Let R be a commutative Noetherian ring and let M be an R–module. In this thesis, we study those R–modules whose Cousin complexes provide Gorenstein injective resolutions. If such a module is finitely generated, then we call that a G–Gorenstein module. It is shown that the class of G–Gorenstein modules strictly contains the class of Gorenstein modules. In this manner, we provide some examples of G–Gorenstein modules over Gorenstein local rings of Krull dimension one, which aren't Gorenstein. Characterizations of G–Gorenstein modules are given and a class of such modules is determined. Also, we provide a Gorenstein injective resolution for a balanced big Cohen–Macaulay R–module. Following, by using of the notion of a G–Gorenstein module, we obtain characterizations of Gorenstein and regular local rings. Next, we study the flat dimension and torsion depth of terms and cokernels of a Cousin complex which is exact at initial terms. This leads us to determine the flat dimension and torsion depth of top local cohomology modules of Cohen–Macaulay (respectively balanced big Cohen–Macaulay) modules. Furthermore, we establish the existence of balanced big Cohen–Macaulay modules over some local rings. Finally, we introduce the notion co–dualizing module.

Introduction

History

All rings considered in this thesis will be commutative and Noetherian and will have non-zero identities; R will always denote such a ring. The Cousin complex is an effective tool in commutative algebra and algebraic geometry. The commutative algebra analogue of the Cousin complex of §2 of chapter IV of Hartshorne [16] was introduced by Sharp in [25]. Then, using the Cousin complex, he characterized Cohen–Macaulay and Gorenstein rings and introduced the theory of Gorenstein modules in [26]. Recall that a non-zero finitely generated R–module M is Gorenstein if the Cousin complex of M with respect to M–height filtration, $C(M)$, is an injective resolution of M. Note that Cohen–Macaulay and Gorenstein rings were characterized in terms of the Cousin complex.

In 1967–69, Auslander and Bridger introduced the concept of G–dimension for finitely generated R–modules. Using this concept, it was proved that the modules having G–dimension zero are Gorenstein projective. It is well-known that G–dimension is a refinement of projective dimension. Finally, in 1993–95, Enochs, Jenda and Torrecillas extended the idea of Auslander and Bridger in [10] and [12], and introduced Gorenstein injective, projective and flat modules (and dimensions), which all have been studied extensively by their founders and by Christensen, Foxby, Frankild, Holm and Xu in [6], [7], [8], [9], [13], [18] and [19].

One of the main open problems in commutative algebra is that of establishing the existence of a big Cohen–Macaulay (balanced big Cohen–Macaulay) module over an arbitrary local ring. The works and writings of Hochster, such as [17], show that, if the existence of such modules could be established, then several conjectures in commutative algebra, some of which are quite long-standing, would be settled. Hochster has established the existence of big Cohen–Macaulay modules whenever the local ring R contains a field as a subring, see [5, Theorem 8.4.2].

Outline of Thesis

The main aim of this thesis is to study the relation between a Cousin complex and Gorenstein injective modules (and dimensions). By linking two above concepts, we introduce the G–Gorenstein modules, those finitely generated R–modules whose Cousin complexes provide Gorenstein injective resolutions.

This thesis contains four chapters. In chapter one, which is preliminaries, we consider some notions that are needed in this thesis.

In chapter two, we study the class of G–Gorenstein modules. In the first section of this chapter, we define the concept of a G–Gorenstein module and provide some characterizations of such modules. The class of G–Gorenstein modules contains the class of Gorenstein modules. We establish some properties of these modules. Also, we show that the class of such modules is closed under finite direct sums. Also, we prove that the G–Gorenstein modules have good behaviour over flat local homomorphisms. In the last section, it is proved that the class of G–Gorenstein modules strictly contains the class of Gorenstein modules. Of course some examples for such modules are provided. Next, using these modules, we characterize finitely generated Gorenstein projective modules in terms of Cousin complexes. Finally, we provide a new characterization for Gorenstein local rings.

In chapter three, we study some invariants of top local cohomology via a Cousin complex. In the first section of this chapter, we give a few results about the torsion functors over a Cousin complex. Using these results, we consider the flat dimension of terms and cokernels of a Cousin complex, whenever it is exact at initial terms. Also, we determine the flat dimension of a top local cohomology module of a balanced big Cohen–Macaulay R–modules which has finite flat dimension. In the second section of this chapter, we study the notion of torsion depth. This study leads us to establish the existence of balanced big Cohen–Macaulay modules over certain local rings. Finally, in the last section,we present the concept of co–dualizing module.

In chapter four, we study the relation between balanced big Cohen–Macaulay modules and Gorenstein (Ω–Gorenstein) injective modules, via the Cousin complex with respect to the dimension filtration.