

# Introduction

The theory of dualizing complexes of Grothendieck and Hartshorne [12, chapter v] has turned out to be a useful tool in commutative algebra, and was studied by R.Y. Sharp and a number of authors, in a series of papers in commutative algebra. The aim of this thesis is to discuss more thoroughly about dualizing complexes. Let us first speak about its motivation. If one decides to study the theory of dualizing complexes for use in commutative algebra, he or she should start with the papers [21], [22], [4], [10], [11] and [24] all (some jointly) written by R.Y. Sharp. In these papers the authors concentrated on establishing the properties of a dualizing complex in the case where it exists. In studying [10] one may fix on the Corollary 4.5 which says that “fundamental dualizing complex for a Noetherian local ring (if exists) is unique up to isomorphism of complexes”. Therefore a natural question arises: *Suppose that  $A$  is a Noetherian local ring which possesses a dualizing complex; so that it possesses “the” fundamental dualizing complex, say  $I^\bullet$ . What is  $I^\bullet$ ?* In other words how one can construct the modules and the morphisms of  $I^\bullet$ . This question is the main motivation of this thesis and is the main object of Chapter 1. Also, in this chapter, we present some applications of our process (see 1.5, 1.6). In particular we will describe the structure of the indecomposable injective modules over an  $(S_2)$  local ring possessing a dualizing complex. We also prove a partial converse to [32, 3.5]. It should be noted that the content of chapter 1 is going to appear in [8].

There are many properties of Cohen-Macaulay rings or modules in the literature. Any Gorenstein ring is a Cohen-Macaulay ring, and any Cohen-Macaulay ring satisfies the Serre condition  $(S_n)$  for all  $n \geq 0$ . Serre condition has many applications in commutative algebra. It is natural to ask what condition on a ring (or on a module) guarantees the condition  $(S_n)$ . Chapter 2 studies this question. Over a Noetherian local ring, in studying the connection between the condition  $(S_n)$  for a finitely generated module  $M$  and a subset of system of parameters for  $M$  of length  $n$

we will find out some stronger condition, which we will call it  $(S'_n)$ . We will answer this question that what happens if  $(S'_n)$  were a localization property. At the end we give some criterions for a module to be  $(S'_n)$  in terms of its Cousin complex with respect to an appropriate filtration.

In this thesis definitions, lemmas, propositions, or theorems which are known before are quoted with a reference afterward; e.g. 1.1.6. Lemma [21, Lemma 3.4]. All the statements which are suppose to be new are just quoted with no references, but followed by proofs.