Asymptotic associated and attached prime ideals related to exact functors

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Summary

Let R be a commutative ring and let P be a projective R-module. Let N (resp.A) denote a Noetherian (resp.Artinian) R-module. An explicit description of the sets $\mathrm{Ass}_R(\mathrm{Hom}_R(P,N))$ and $\mathrm{Att}_R(\mathrm{Hom}_R(P,A))$ are given. As a consequence several results concerning asymptotic behavior of the exact functor $\mathrm{Hom}_R(P,\cdot)$ are deduced.

Then, we develop the theory of coassociated prime ideals in some aspects. Also we provide an affirmative answer to a question raised by L.Melkersson in a epical case (see [**17**]).

In the next part of the thesis, for a flat R-module F, the set $\mathrm{Att}_R(F \otimes_R A)$ is determined precisely. Moreover in the case that R is Noetherian , the set $\mathrm{Att}_R(\mathrm{Hom}_R(F,A))$ is described. This gives a complete answer to Ansari-Toroghy's question (see [**2**]). As an immediate application a criteria for the non-vanishing of certain local cohomology modules is given.

At the end , we extend our previous considerations regarding asymptotic primes to the general situation. To this end, let I be an ideal of R. Let $\mathcal{C}_R$ denote the category of R-modules and R-homomorphisms and $\mathcal{C}_\mathcal{N}$ (resp.$\mathcal{C}_\mathcal{A}$) denote the subcategory of Noetherian (resp. Artinian) R-modules. For a linear exact covariant (resp.contravariant) functor $T : \mathcal{C}_\mathcal{N} \longrightarrow \mathcal{C}_R$, it is shown that both sequences of sets $\mathrm{Ass}_R(\frac{T(N)}{I^nT(N')})$ and $\mathrm{Ass}_R(\frac{I^nT(N)}{I^nT(N')})$ (resp. $\mathrm{Att}_R(T(N/N' :_{T(N)} I^n)$ and $\mathrm{Att}_R(T(N/N') :_{T(N)} I^n/0 :_{T(N)} I^n))$ are eventually constant for large n. Also, the dual results are shown to be true for a linear exact functor $T : \mathcal{C}_\mathcal{A} \longrightarrow \mathcal{C}_R$. These results are far reach generalizations of the corresponding results in [**5**], [**29**], [**30**], [**27**], [**19**], [**20**], [**22**] and [**12**].