Talk 1: Locally finite dimensional representations over Noetherian algebras

Talk 2: Locally finite dimensional representations over Noetherian Hopf algebras

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Shortly after Eckmann and Schopf's definition of the concept of an injective modules, Matlis showed that the injective hull of a simple module over a commutative Noetherian ring R is Artinian. Vamos showed that the latter condition is actually equivalent to the commutative ring R being locally Noetherian. However, injective hulls of simple modules over non-commutative Noetherian rings might not be Artinian. Slightly different, but related, one might consider Noetherian algebras A over a field F and ask whether any finitely generated submodule of the injective hull of a finite dimensional simple A-module need to be finite dimensional. In other words, is the category of locally finite dimensional representations of A closed under taking injective hulls. This is certainly the case for commutative algebras, but not for non-commutative algebras. Feldvoss proved, using results of Donkin and Dahlberg, that the category of locally finite dimensional representations over an enveloping algebra $U(g)$ of a finite dimensional Lie algebra g is closed under taking injective hulls if and only if g is a solvable Lie algebra. Furthermore, the group ring $F[G]$ of a polycyclic-by-finite group also has this property. Having these motivating examples in mind we intend to find necessary and sufficient conditions for the category of locally finite dimensional representations over a Noetherian algebra A over a field F to be closed under taking injective hulls.

In the first talk we will characterize Noetherian algebras A whose category of locally finite dimensional representations is essentially closed through their finite dual coalgebra

$$A^\circ = \{f \in Hom(A,F) : \mathrm{Ker}(f) \text{ contains an ideal of finite codimension}\}$$

and provide sufficient conditions that involve the maximal ideals of A of finite codimension. We then consider Ore extensions of an algebra A, which are non-commutative polynomial rings in a variable x such that the coefficients in A obey to

$$xa = \sigma(a)x + \delta(a), \qquad \forall a \in A$$

for a given automorphism σ and a σ-derivation δ of A. As an important tool we will use the Rees ring of an ideal in the Ore extension and find condition under which it is Noetherian.

In the second talk we will provide some background on the theory of Hopf algebras, having in mind that enveloping algebras of Lie algebras and group algebras are fundamental example. We will show that the question of the category of locally finite dimensional representations of a Noetherian Hopf algebra to be essentially closed can be reduced to the study of finitely generated extensions of the trivial representation. Our main results will show that Ore extension of commutative Noetherian Hopf algebras, certain Hopf crossed product algebra $A\#_\sigma H$ and all affine Hopf algebra domains of Gelfand-Kirillov dimension ≤ 2 with $Ext^1_H(F,F) \neq 0$ are Noetherian algebras whose category of locally finite representations is essentially closed.

Throughout the talks we will assume that the base field is algebraically closed and has characteristic zero.

The talks will be based on the preprint "Locally Finite Representations Over Noetherian Hopf algebras" jointly written with my coauthor Can Hatipoglu. A PDF of this preprint can be found on ArXiv: `https://arxiv.org/abs/2010.14192`