

## COHOMOLOGY THEORIES FOR MODULES OF FINITE G-DIMENSION

L.L. Avramov

Over a ring  $R$ , Noetherian on both sides, we study finitely generated modules  $M$  that have finite G-dimension in the sense of Auslander and Bridger. Besides the *absolute cohomology*  $Ext_R^n(M, \_)$ , two other theories are defined for such modules. In *relative cohomology*  $Ext_{\mathcal{G}}^n(M, \_)$  the modules of G-dimension 0 take the role of projectives. *Tate cohomology*  $\widehat{Ext}_R^n(M, \_)$  allows  $M$  to be replaced by any of its syzygies. Comparison morphisms  $\varepsilon^n: Ext_{\mathcal{G}}^n(M, \_) \rightarrow Ext_R^n(M, \_)$  and  $\widehat{\varepsilon}^n: Ext_R^n(M, \_) \rightarrow \widehat{Ext}_R^n(M, \_)$  link these theories. We establish basic properties of relative and Tate cohomology, and embed the comparison morphisms into a canonical long exact sequence of functors  $Ext_{\mathcal{G}}^n(M, \_) \rightarrow Ext_R^n(M, \_) \rightarrow \widehat{Ext}_R^n(M, \_) \rightarrow Ext_{\mathcal{G}}^{n+1}(M, \_)$  starting with a monomorphism  $\varepsilon^1$ . We show that these results provide efficient tools for computation over commutative local rings. This lecture is based on joint work with Alex Martsinkovsky. Some of these results have been generalized to modules of finite Gorenstein projective dimension by Oana Veliche.

## GROWTH OF (CO)HOMOLOGY OVER COMPLETE INTERSECTION RINGS

L.L. Avramov

We develop geometric methods for the study of finitely generated modules  $M, N$  over a *local complete intersection*  $R$ . A cone, that is, a homogeneous algebraic set  $V_R^*(M, N)$  is attached to each such pair and is used to study their homological invariants, such as  $Ext_R^n(M, N)$  and  $Tor_n^R(M, N)$ . The dimension of this variety measures the rate of growth of the modules  $Ext_R^n(M, N)$  as  $n$  goes to infinity. This point of view allows us to show, in particular, the unexpected result that  $Ext_R^n(M, N) = 0$  for all  $n \gg 0$  if and only if  $Ext_R^n(N, M) = 0$  for all  $n \gg 0$ , if and only if  $Tor_n^R(M, N) = 0$  for all  $n \gg 0$ . This lecture is based on joint work with Ragnar Buchweitz.

## VANISHING OF (CO)HOMOLOGY OVER GORENSTEIN RINGS

L.L. Avramov

The symmetry of vanishing of  $Ext$  in both module variables over complete intersection rings raises the question whether this property characterizes the class of Gorenstein local rings. Recent progress on this still open question by Craig Huneke, David Jorgensen, and Liana Şega will be reported. Another characterization of Gorenstein rings has been conjectured, in terms of vanishing of  $Ext_R^n(D, R)$  for all  $n \geq 1$  when  $D$  is a dualizing complex. Joint work with Buchweitz and Şega will be reported.