MEASURING GORENSTEIN DIMENSIONS

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The classical G–dimension was introduced by M. Auslander in [1]. It is a *refinement* of the projective dimension for f.g. modules M over a commutative Noetherian ring R. That is, there is always an inequality $\text{G–dim}_R\, M \leq \text{pd}_R\, M$, and equality holds when $\text{pd}_R\, M < \infty$.

When finite, the G–dimension of a module M can be measured by non-vanishing of Ext-modules of M against R. I.e., if $\text{G–dim}_R\, M < \infty$ then

$$\text{G–dim}_R\, M = \sup\{m \in \mathbb{Z} \mid \text{Ext}_R^m(M,R) \neq 0\}.$$

The G–dimension of M is, of course, defined in terms of resolutions of M by certain modules. And these certain modules are exactly those that arise as cokernels in *complete finite free resolutions*, i.e. exact sequences of f.g. free moduules which stay exact under dualization. In view of this, the above measure is quite intuitive.

In [4] and [5] E.E. Enochs et al. introduced Gorentein projective, flat, and injective modules. These also arise as kernels and cokernels of certain complete resolutions, and similar intuitive measures of Gorenstein projective, flat and injective dimensions arise.

Apart from these generic measures of the Gorenstein dimensions, a host of other measures apply. Over local Cohen–Macaulay rings the varaity is particularly rich, and this was explored in some depth in [2]. The talk will provide an overview of these measures and discuss relations the restricted homological dimensions studied in [3].

References

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