The Chromatic Number of Graphs on Surfaces

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Graph coloring is a extensively studied subject, partly because of its relation to optimization (time table problems). One of the main sources of inspiration was the 4 Color Problem (now a theorem). In 1890 Heawood considered the analogue for higher surfaces. This problem, known as the Heawood map color theorem, was settled by Ringel and Youngs in 1968. For example, the number of colors needed in the projective plane and the Klein bottle is 6. For the torus it is 8, etc. Although these numbers tend to infinity, there is a 5-color theorem for each surface in the following sense: For every surface S, there exist a finite number of (forbidden) graphs such that an arbitrary graph on S can be 5-colored if and only if it does not contain one of the forbidden graph as a subgraph. There is no 4-color theorem of this type. In the talk these and related results will be discussed.

The Chromatic Polynomial of a Graph.

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The chromatic polynomial P(G,k) of a graph G is the number of colorings of G with k available colors. The chromatic polynomial was introduced by Birkhoff in 1912 in order to study the 4-Color Problem. Although the chromatic polynomial has not been very successful for coloring problems, it has been studied for several other reasons, primarily by mathematicians but also by physicists. In this talk, basic properties of the chromatic polynomial will be discussed, including a recent connection to the Hamiltonian cycle (travelling salesman's) problem.