The Chromatic Number of Strongly Regular Graphs

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In 1970 A.J. Hoffman published the following lower bound for the chromatic number $\chi(\Gamma)$) of a graph Γ.

$$\chi(\Gamma) \geq 1 - \lambda_1/\lambda_n.$$

Here $\lambda_1 \geq \ldots \geq \lambda_n$ are the eigenvalues of the adjacency matrix of Γ. This bound still is one of the major algebraic tools to estimate the chromatic number. About 10 years later the author strengthened Hoffman's approach a bit and showed among other that

$$\chi(\Gamma) \geq \min\{m_n, 1 - \lambda_n/\lambda_2\},$$

where m_n is the multiplicity of λ_n. This bound works well for strongly regular graphs, and made it possible to determine all such graphs with chromatic number at most 4. Recently this work has been continued with the study of 5-chromatic strongly regular graphs. Another interesting class are the strongly regular graphs for which Hoffman's bound is tight. These are related to association schemes, linked symmetric 2-designs and Bush type Hadamard matrices.

Conditions for Singular Incidence Matrices

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Suppose one looks for a square integral matrix N, for which $NN^\top$ has a prescribed form. Then the Hasse-Minkowski invariants and the determinant of $NN^\top$ lead to necessary conditions for existence. The Bruck-Ryser-Chowla theorem gives a famous example of such conditions in case N is the incidence matrix of a square block design. This approach fails when N is singular. In this paper it is shown that in some cases conditions can still be obtained if the kernels of N and $N^\top$ are known, or known to be rationally equivalent. This leads for example to non-existence conditions for self-dual generalised polygons, semi-regular square divisible designs and distance-regular graphs.