Matrix Completion Problems

Charles R. Johnson
Department of Mathematics
College of William and Mary
USA

By a partial matrix, we mean a rectangular array, in which some entries are specified, while the remaining, unspecified, entries are free to be chosen from an agreed upon set, such as an indicated field. By a completion of a partial matrix, we mean an allowed choice of values for the unspecified entries, resulting in a conventional matrix. A matrix completion problem then asks for which partial matrices does there exist a completion of a desired type, such as from a class of matrices of interest (eg positive definite, given rank, a P-matrix, etc). The patterns for the specified entries that guarantee a completion when obvious necessary conditions are met are usually a key starting point, and what can be said about the set of completions may also be of interest. A general theory for the existence of solutions to a matrix completion problem will be outlined, and then we survey a variety of results and techniques for matrix completion problems involving familiar classes of matrices, such as the positive definite matrices, contractions, totally nonnegative matrices, etc (time permitting).

Eigenvalues, Eigenvectors, Graphs and Multiplicities

Charles R. Johnson
Department of Mathematics
College of William and Mary
USA

Let G be an undirected graph on n vertices, and let S(G) denote the collection of all n-by-n real symmetric matrices whose graph is G. The diagonal entries of matrices in S(G) are not constrained by G (but they are real). We are interested in the possible lists of multiplicities for the eigenvalues among the matrices in S(G). This has two interpretations; the unordered multiplicities indicate the multiplicities that occur, without respect to the numerical values of the underlying eigenvalues, while the ordered multiplicities are listed in the numerical order of the underlying eigenvalues. We survey current results about both kinds of multiplicities, for certain classes of graphs, especially for certain trees. Among a number of major questions is whether the problem of ordered multiplicities is equivalent to the inverse eigenvalue problem.