Binomial Identities and Large Sets of t-Designs

B. Tayfeh-Rezaie
School of Mathematics
Institute for studies in theoretical Physics and Mathematics (IPM)
Iran

A *simple* t-(v, k, λ) design is a subset of the set of all k-subsets of a v-set such that every t-subset of the v-set is exactly contained in λ elements. A large set of t-(v, k, λ) designs of size N, denoted by $\mathrm{LS}[N](t, k, v)$, is a partition of all k-subsets of a given v-set into N disjoint t-(v, k, λ) designs, where $N = \binom{v-t}{k-t}/\lambda$. There are many binomial identities which have set theoretic interpretations. Perhaps the most simple ones are

$$\binom{v}{k} = \binom{v}{v-k},$$

$$\binom{v}{k} = \binom{v-1}{k} + \binom{v-1}{k-1}.$$

The first identity holds since there is a one to one correspondence between the sets of all k-subsets and $(v-k)$-subsets of a v-set. The second one is true as the set of all k-subsets of a v-set can be partitioned into two parts: Fix a point x and consider the set of all k-subsets of the remaining $v-1$ points as one part and the set of all $(k-1)$-subsets with x attached to each of them as the other part. These identities and their interpretations by set partitions provide two recursive constructions for large sets. The first one states that if there exists a $\mathrm{LS}[N](t, k, v)$, then there exist a $\mathrm{LS}[N](t, v-k, v)$. The other is that if there exist $\mathrm{LS}[N](t, k, v-1)$ and $\mathrm{LS}[N](t, k-1, v-1)$, then there exists $\mathrm{LS}[N](t, k, v)$. In this talk we will discuss a few more complicated identities and the recursive methods of construction which can be obtained from them.