INVERSE PROBLEMS FOR PARABOLIC EVOLUTION EQUATIONS (I, II & III)

R. CONT

This presentation is concerned with some model identification problems in which the coefficient of a parabolic evolution equation is to be retrieved from the observation of the solution at a (finite) set of points. Examples are diffusion equations:

$$\frac{\partial u}{\partial t} = \frac{\sigma^2(t,x)}{2}\frac{\partial^2 u}{\partial x^2} + b\frac{\partial u}{\partial x} \quad u(0,x) = u_0(x). \tag{1}$$

where $\sigma(.,.)$ is unknown and integro-differential equations such as

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + b\frac{\partial u}{\partial x} + \int dy\nu(y)\ [u(t,x+y) - u(t,x) - y\frac{\partial u}{\partial x}(t,x)] \quad u(0,x) = u_0(x). \tag{2}$$

where the unknown coefficient to be identified is the kernel $\nu(.)$. Via the Feynman-Kac formula, these equations also admit a probabilistic interpretation through their relation with continuous-time Markov processes and can be interpreted as moment problems for a certain class of Markov processes. They arise in the description of physical systems subjected to noise and also in the context of *option pricing theory* in the mathematical modeling of financial markets.

Identifying $\sigma(.,.)$ or $\nu(.)$ from observations $(u(t_i, x_i), i = 1..n)$ is a non-linear inverse problem which is ill-posed. First we discuss the theoretical solution of the inverse problem in the case of complete data, discuss its ill-posedness and its ineffectiveness as a numerical algorithm. In the case of finite/ incomplete observations, we then propose several methods for retrieving solutions in a stable manner. The first approach is based on Tikhonov regularization using a smoothness norm as penalization term. The second algorithm uses the probabilistic interpretation of the evolution equation and uses the relative entropy as a penalization term. The third algorithm recasts the problem as a constrained stochastic control problem which is solved through a Hamilton-Jacobi-Bellman equation. The last approach avoids penalization methods through a random search in the parameter space using an evolutionary (genetic) algorithm.

In each case we describe the numerical algorithms involved, comparing their performance and discussing implementation issues.

Outline of lectures:

1. Inverse parabolic problems.
2. Solution by Tikhonov regularization.
3. Solution by entropic penalization.
4. Solution via stochastic control.
5. Solution via evolutionary algorithms.