X-RAY TOMOGRAPHY

F. Natterer

X-ray tomography is still the most prominent example of an inverse problem. Studying x-ray tomography is still the best way to become familiar with inverse problems. We describe the principles of x-ray tomography and its mathematical modelling. Based on x-ray tomography we discuss the basic questions for each inverse problem: Uniqueness, stability, and resolution. These three topics can be dealt with for x-ray tomography in a an elementary and rigorous way. By elementary we mean that the audience is familiar with elementary functional analysis (Hilbert spaces, linear operators, inverses, ranges) and elementary Fourier analysis.

RECONSTRUCTION ALGORITHMS

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We discuss in depth the filtered backprojection algorithm, which is the most widely used algorithm in clinical applications. We study its resolution in the light of the sampling theorem, its stability and efficiency. We also consider its recent extensions to 3D tomography. A second class of algorithms we discuss is the class of iterative methods that are based on the Kaczmarz method. In particular we investigate the speed and the qualitative behavior of convergence.

NONLINEAR TOMOGRAPHY

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In this section we consider innovative tomographic techniques, such as transmission ultrasound, laser, and impedance tomography, that are not yet used in practice. These techniques require the solution of nonlinear problems. They are only remotely related to X-ray tomography, but the basic principles, such as backprojection, are still valid. We show that all these problems can be dealt with by a nonlinear extension of the Kaczmarz method, in conjunction with highly efficient methods for solving PDE's.