Variations on the Lefschetz Principle (Presented by Title)

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Some sixty years ago, Tarski isolated the basic modeltheoretic content of Lefschetz's Principle in algebraic geometry, and this is now presented in the first pages of any text on modeltheoretic algebra as an easy consequence of quantifier elimination for algebraically closed fields. By now most logicians are familiar with the beautiful consequence derived by Ax in the late 1960's, namely that injective endomorphisms of varieties are surjective, proved by reducing to the case of the algebraic closure of finite fields, and then using a primitive counting argument. I will trace the ensuing elaborations of these basic ideas in two parts.

In the first part, the following are discussed: Topological interpretation of the basic principle. This connects the Tarski spaces of types and the space Spec(Z) of algebraic geometry. Formulations in terms of ultraproducts and sheaves. Ax's work on finite fields. Type spaces, and the Vietoris topology on the absolute Galois group of the field of rationals. p-adic fields. Various interpretations of the operation "let p tend to 0".

The second part is devoted to: The Lefschetz Fixed Point Theorem in model theory, and the model theory around the Weil Conjectures. The Frobenius, and the Witt Frobenius, and their logic. Cohomology and model theory.

Update as of October 13:

Due to cancelation of presence of Prof. Macintyre at the meeting, the second hour is now devoted to short presentations by some of the other invited speakers and maybe of the organizers reporting on some aspects of the works of Angus Macintyre in Mathematical Logic and possibly their collaborations with him. In the first hour, a related activity will be held.