Arithmetic Complexity, Non-standard Rings of Integers, and Irrationality

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The first part of this work will explicitly characterize the one-variable arithmetic functions that are computable in a uniformly bounded number of steps using certain arithmetic operations as given. This characterization is deduced from algebraic properties of certain non-standard rings of integers.

The second part deals with lower bounds on the arithmetic complexity of computing other arithmetic functions. A sample result is:

Any algorithm computing the greatest common divisor and using addition, subtraction, and integer division with remainder as given, will take more than $(1/7)\log\log\ a$ steps on infinitely many integer inputs (a, b) with $a > b > 1$.

It turns out that classical facts like the irrationality of the square root of two and the transcendence of e are highly relevant in this connection.

The above represents joint work with Yiannis Moschovakis.