An IP-Sensitivity Result for Density Mod 1 and a General Liouville's Theorem

S.M. Ayat and Mojtaba Moniri
IPM and Tarbiat Modarres University
Tehran, Iran

We first present an example of a real closed field with two integer parts $\langle F, I_1, I_2\rangle$, where for some $\alpha \in I_1$, the set $\{u\alpha - \lfloor u\alpha \rfloor_{I_2} | u \in I_2\}$ is dense in $[0,1)_F$. Independent of this, for the general $\langle F, I_1, I_2\rangle$ context of an ordered field with two integer parts, we show that if I_1 models Open Induction, then so does I_2.

In the second part of this work, we establish a general Liouville's theorem (on approximation-exponents) for $\langle F, I\rangle$, an ordered field with an IP. Finally, assuming that they are of countable cofinality, we show that the Scott completion of F is transcendental over the fraction field of I.