

Real Closed Fields and the Absolute Existence of Irrationals

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This is in sequel to part of the joint work with Seyed Masih Ayat contributed to the same conference, from the viewpoint of the general matter of IP-sensitivity or insensitivity of real closed fields to satisfy a classical standard fact: here we deal with the existence of irrationals. We show that all real closed fields of infinite transcendence degree (over the rationals) have IP's with respect to which there remains (a necessarily dense set of) irrationals. It is known on the other hand, by an early observation of van den Dries, that some such fields also have IP's not leaving anything irrational. We present non-Archimedean real closed fields which have IP's with primes whose powers are cofinal. Such IP's therefore leave the square roots of those primes irrational. This includes certain finite transcendence degree cases not covered by the above result. We will study whether there are non-Archimedean real closed fields *with absolute existence of irrationals* and indeed whether in the finite transcendence degree case there are any failing that.