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Diffusion and Elasticity Problems Posed in a Cylinder of Small Diameter with the Dirichlet Boundary Condition on a Very Small Part of One of its Extremities

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We consider a cylinder of fixed length whose diameter ε tends to zero. In this thin cylinder we consider successively a conductivity problem and a linearized elasticity problem, both for anisotropic and heterogeneous materials. The Dirichlet boundary condition is imposed on the whole of one of the extremities of the cylinder, but it is imposed on only a very small part of size $\varepsilon r^\varepsilon$ (where r^ε tends to zero) of the other extremity. On the remainder of the boundary of the thin cylinder one has the Neumann boundary condition.

In the conductivity case, there is one critical size, namely $r^\varepsilon \approx \varepsilon$, which separates two other regimes: when $r_\varepsilon \ll \varepsilon$, the region where the Dirichlet boundary condition is imposed is so small that this condition disappears, and one has the Neumann boundary condition at the limit; when $\varepsilon \ll r_\varepsilon$, the region where the Dirichlet condition is imposed is sufficiently large to imply that one has the Dirichlet boundary condition at the limit; finally when $r_\varepsilon = \rho\varepsilon$, one has at the limit a Fourier boundary condition, with a coefficient which depends on ρ and ensures a transition of the boundary condition when ρ tends to zero and to infinity.

In the elasticity case, there are 3 critical sizes, namely $r^\varepsilon \approx \varepsilon^3$, $r^\varepsilon \approx \varepsilon$, and $r^\varepsilon \approx \varepsilon^{1/3}$, which separate 4 other regimes.