Negative Gaussian Curvature, Hyperbolic Monge-Ampère Equations, and Systems in Riemann Invariants with Big Coefficients: Regular Solutions on the Whole Plane

Yu.N. Bratkov
Department of Mathematics
Moscow State University
Moscow, Russia

The hyperbolic equation

$$hess\ z = -k^2(x,y)(1 + z_x^2 + z_y^2)^2 \tag{1}$$

is considered. Here, hess $z = z_{xx}z_{yy} - z_{xy}^2, k = \bar{k}(x,y) + \varepsilon(x,y), \bar{k}(x,y) = \frac{1}{1+x^2+y^2}$. The existence of a C^3-solution on the whole plain is proved. The sufficient conditions are formulated.

The function $-k^2(x,y)$is Gaussian curvature of the surface $z = z(x,y)$. For the surface $z = \frac{1}{2}(x^2 - y^2)$ we have $k = \bar{k}$.

The equation (1) is reduced to some system in Riemann invariants with big coefficients. "Big" means $O(\frac{1}{x})$, $x \to \infty$, i.e. $\int_{-\infty}^{+\infty} = \infty$. Systems in Riemann invariants with small coefficients are studied in [1], with the correspondent Monge-Ampère equations. The result is the next.

The problem for the hyperbolic Monge-Ampère equation

$$\begin{cases} A + Bz_{xx} + Cz_{xy} + Dz_{yy} + \text{hess } z = 0, \\ z(0,y) = z^\circ(y),\ z_x(0,y) = p^\circ(y),\ y \in \mathcal{R} \end{cases} \tag{2}$$

is considered. A, B, C, D depends on x, y, z, z_x, z_y. The equation is hyperbolic when $C^2 - 4BD + 4A > 0$. The existence of a unique C^3-solution on the whole plain is proved. The sufficient conditions are formulated.

Regular solutions of the hyperbolic Monge-Ampère equation

$$\text{hess } z = -f^2(x,y) \tag{3}$$

on the whole plain were considered by J.X. Hong [2].

The unexistence of a complete surface with $-k^2(x,y) \leq$ const< 0 is the Hilbert-Efimov's theorem [3,4]. The positive J.X.Hong's result [5] is the existence of a complete surface, when k^2 is decreasing more than $\rho^{-(2+\delta)}, \rho$ is the distance from a point. Thus, the proposed result (1) isnt new in geometry (if Hong's proof is correct). It is new in

PDE. Also, our world isn't $\mathcal{R}^3$ or Minkowski space [6,7]. This old geometric problem (Lobachevski, Beltrami, Hilbert) seems to be non-actual now.

To build regular solutions on the whole plain is an extremal kind of sport. If a domain isnt the whole plain, it is possible to move singularities outside the domain. Really it is the usual technic. It is impossible to move out when the domain is the whole plain. Thus, this is entirely another problem.

References

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