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Application of the Semidefinite Programming in Quantum Information

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Over the past years, semidefinite programming (SDP) in particular, has come to be recognized as valuable numerical tools for control system analysis and design. In (SDP) one minimizes a linear function subject to the constraint that an alline combination of symmetric matrices is positive semidefinite. Also semidefinite programming has a nice duality structure, with, the associated dual program. We can use the (SDP) to obtain the different aspects of the quantum information as follows:
In general the calculation of robustness of entanglement for the mixed entangled quantum states is rather difficult to handle analytically. Using the convex semi-definite programming method, the robustness of entanglement of some mixed enlarged quantum states such as: $2\otimes 2$ Bell decomposable (BD) states, a generic two qubit state in Wootters basis, iso-concurrence decomposable states, $2 \otimes 3$ Bell decomposable states, $d \otimes d$ Worner and isotropic states , a one parameter $3 \otimes 3$ state and finally multi partite isotropic state, is calculated exactly. Also, an analytic expression is given for separable states that wipe out all entanglement and it is further shown that they are on the boundary of separable states. On the other hand, using semidefinite optimization method, we prove the Lewenstein-Sanpera lemma in a simple elegant manner. As we know the usual current methods for obtaining the optimal Lewenestein Sanpera decomposition of a mixed state are difficult to handle analytically. We provide a simple analytical expression for optimal Lewenstein Sanpera decomposition for some examples such as: Bell decomposable state, Iso-concurrence state, generic two qubit state in Wootters's basis, $2 \otimes 3$ Bell decomposable state, $d \otimes d$ Werner and isotropic states, a one parameter $3 \otimes 3$ state and finally multi partite isotropic state. In the subsequent example using the semidefinite programming or convex optimization method, the minimum relative entropy of Bell decomposable states has been evaluated and it is shown that the separable states that minimize relative entropy are the same as the states which optimize the Lewenstein-Sanpera decomposition. Finally, using the SDP method we obtain properties of frames and basis in Hilbert space. Also, with SDP we obtain the projection method in frame operator based on basis, biorthogonal basis and frames.