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Parabolic Equations Modeling Chemotaxis

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We will present some useful techniques for the analysis of the Patlak, Keller and Segel model, which accounts for cell movement. We make the hypothesis that cells respond to a chemical signal c. Then cell density n satisfies an equation of advection-diffusion.

$$\begin{cases} \partial_t n = \nabla.(\nabla_n - \chi n \nabla_c) = 0 \quad t \geq 0, x \in \Omega, \\ \Gamma \partial_t c = \Delta c + n - \alpha c. \end{cases}$$

We will consider only the case of a domain $\Omega \subset \mathbb{R}^2$.

The behaviour of this system depends on the parameter χM (M is the total mass of cells): if $\chi M < \underline{c}$ then solution exists for all time and remains bounded; whereas if $\chi M > \overline{C}$ then solution blows up in finite time. We will expand tools which lead to this result in particular cases (basically Ω is either a bounded domain or the whole space).

- *A priori* estimates.

- Energy methods.

- Blow up of solutions in the whole space or in the radial case.