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A Complete Formulation of Baum-Connes' Conjecture for the Action of Discrete Quantum Groups

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We formulate a version of Baum-Connes' conjecture for a discrete quantum group, building on our earlier work ([9]). Given such a quantum group $\mathcal{A}$, we construct a directed family $\{\mathcal{E}_F\}$ of C^*-algebras (F varying over some suitable index set), borrowing the ideas of [5], such that there is a natural action of $\mathcal{A}$ on each $\mathcal{E}_F$ satisfying the assumptions of [9], which makes it possible to define the "analytical assembly map", say $\mu_i^{r,F}$, $i = 0, 1$, as in [9], from the $\mathcal{A}$-equivariant K-homolgy groups of $\mathcal{E}_F$ to the K-theory groups of the "reduced" dual $\hat{\mathcal{A}}_r$. As a result, we can define the Baum-Connes' maps $\mu_i^r : \overset{\lim}{\longrightarrow} KK_i^{\mathcal{A}}(\mathcal{E}_F, \mathbb{C}) \to K_i(\hat{\mathcal{A}}_r)$, and in the classical case, i.e. when $\mathcal{A}$ is $C_0(G)$ for a discrete group, the isomorphism of the above maps for $i = 0, 1$ is equivalent to the Baum-Connes' conjecture. Examples are discussed.

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