Noncommutative Instantons

(3 Lectures)

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These lectures are devoted to recent work on deformation of spaces, in particular spheres, and deformation of vector bundles which are relevant in physics, notably monopole and instanton bundles.

A first class of examples consists of toric noncommutative manifolds which were introduced out of cycles in homology. These examples have non-trivial global features and can be endowed with a structure of noncommutative manifolds, in terms of a spectral triple. In particular, noncommutative spheres are isospectral deformations of usual spherical geometries. Instantonic bundles and Yang-Mills duality equations can be introduced and a geometric analysis paralleling the one on commutative space can be performed.

A second class of examples is made of spaces coming from quantum groups, some of which can again be endowed with a spectral triple. Also one can construct generalizations of instanton bundles.

Finally, we also review some work on harmonic maps and non-linear sigma-models in noncommutative geometry, with particular attention on models on noncommutative tori.