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WDM and Directed Star Arboricity of Digraphs

O. Amini
Ecole Polytechnique
Paris, France

A *k-directed-star-colouring* of a digraph is a colouring of the arcs of D such that on each vertex two incoming arcs have different colours and in addition an incoming arc has always a colour different from an outgoing arc. The *directed-star-arbroicity* of a digraph D, denoted by $dst(D)$, is the minimum integer k such that there exists a k-directed-star-colouring (For instance orientations of cycles have $dst = 2$ apart from directed odd circuits.) This notion was defined by Guiduli in [4] and is an analog of the *star-arboricity* defined by Algor and Alon in [1].

The problem we are interested in is related to WDM (Wavelength Division Multiplexing) in star optical network, see [3]. We are given a star network in which a center node is connected to a set of nodes V. Each node v sends a set of $s(v)$ multicasts to the sets of nodes $S_1(v), ...S_{s(v)}(v)$. The central node redirect an arriving message toward other vertices on the same frequency. We study the general problem of wavelength assignment in this kind of networks, taking into account the interferences. The case of one multicast reduces to a *k-directed-star-colouring*.

For $\Delta^- \leq \ell$ we prove $dst \leq 2\ell + 1$. This fills the gap between the bounds given in [3]. In the more general case where each edge of the star network has g fibers and we have n multicasts, we give almost tight bounds for number of colours in terms of (Δ^-, n and g). To obtain these bounds we use Lovász Local Lemma, decomposition into outbranchings techniques and minimal feedback arc set.

This is a joint work with F. Havet, F. Huc and S. Thomasse.

References

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