

Cospectral Graphs and the Generalized Adjacency Matrix

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Consider a graph Γ with adjacency matrix A . Any matrix of the form $\alpha A + \beta J + \gamma I$ with $\alpha \neq 0$ is a *generalized adjacency matrix* of Γ . Without loss of generality we take $\alpha = 1$ and $\gamma = 0$. An old result of Johnson and Newman states that if two graphs are cospectral with respect to $A + \beta_1 J$ and $A + \beta_2 J$ ($\beta_1 \neq \beta_2$), then they are cospectral with respect to all generalized adjacency matrices. In the talk we will focus on graphs cospectral with respect to just one value of β . We will construct such pairs for all $\beta \in \mathbb{R}$. More in particular, for all rational $\beta \in (-1, 0)$ we will construct cospectral pairs, where one is regular, and the other one is not. We'll prove that this phenomenon is impossible for other values of β , and find along the way a short proof of the mentioned theorem by Johnson and Newman.