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The Number of Latin Squares

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The problem of counting Latin squares was studied by Euler more than 220 years ago and interest has not declined since then.

Two types of enumeration are involved, which can be called *labelled* and *unlabelled*. In labelled enumeration, each Latin square is regarded as distinct from all others. The most recent result, by the speaker and Ian Wanless, is that the number of Latin squares of order 11 is 776966836171770144107444346734230682311065600000.

In the case of unlabelled enumeration, various definitions of equivalence are defined and one tries to determine the number of equivalence classes. The equivalence called *isotopy* is defined by saying that two Latin squares are isotopic if one can be obtained from the other by permuting the rows, the columns, and the symbols. In the case of *paratopy*, we also allow interchange of the three roles "row", "column" and "symbol". The most recent result is by the speaker together with Alison Meynert and Wendy Myrvold, that the number of isotopy classes of order 10 is 208904371354363006 and the number of paratopy classes (also called *main classes*) of order 10 is 34817397894749939.

We will explain how these results were obtained, and also mention the related problems of enumerating finite quasigroups and loops. Finally, we report on the search for three mutually orthogonal Latin squares of order 10.