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More on Decompositions of Edge-colored

R. M. Wilson
California Institute of Technology
California, USA

Given a simple graph G with m edges, let d be the greatest common divisor of the degrees of its vertices. With finitely many exceptions, the edge set of the complete graph K_n on n vertices can be partitioned into (edge-disjoint but not necessarily vertex-disjoint) copies of G provided that $n(n-1) \equiv 0 \pmod{2m}$ and $n-1 \equiv 0 \pmod{d}$. This is a result of this writer from 1975.

We are concerned with extensions of this result. Many combinatorial problems can be seen to be equivalent to certain decompositions of edge-colored complete graphs, $K_n^{(\lambda_1,\ldots,\lambda_r)}$ in which any two distinct vertices are joined by λ_i edges of color i, $i = 1, 2, \ldots, r$. In general, we are given a family $\mathcal{G}$ of graphs whose edges are colored with elements of $\{1, 2, \ldots, r\}$ and we wish to find a family of subgraphs of $K_n^{(\lambda_1,\ldots,\lambda_r)}$, each isomorphic to a member of $\mathcal{G}$, so that each edge of the complete graph is in exactly one of the subgraphs.

For the case when the graphs in $\mathcal{G}$ are simple (i.e. have at most one edge of color i joining any two disitinct vertices, $i = 1, 2, \ldots, r$), necessary and asymptotically sufficient conditions on n in term of $\mathcal{G}$ were given by E. Lamken and the writer in 2000. At the time, we thought this would be sufficient for all reasonable applications to problems in design theory, but it became apparent that it would be useful to have results for the case when the graphs in $\mathcal{G}$ are not necessarily simple.

We can give an answer (necessary and asymptotically sufficient conditions for the existence of a decomposition) in the general case, but it is much more complex, computationally. This is joint work with Anna Draganova and Yukiyasu Mutoh. Recently, we have extended our results to directed edge-colored graphs.

We will give examples of combinatorial problems that can be put into this framework, give a precise description of the main result, and briefly discuss certain points of the proof.