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Modular Ranks and Smith Normal Forms of Some Incidence Matrices

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In this talk, we will survey some recent results on p-ranks and Smith normal forms of some incidence matrices from finite geometry.

Let V be an $(n+1)$-dimensional vector space over $\mathrm{GF}(q)$, where $q = p^t$, p is a prime. For $1 < r \leq n$, let $A^n_{1,r}(q)$ be the (0,1)-incidence matrix with rows and columns respectively indexed by the r-and 1-dimensional subspaces of V, and with (X, Y)-entry equal to one if and only if the 1-dimensional subspace Y is contained in the r-dimensional subspace X. We will discuss recent work on the complete determination of the Smith normal form of $A^n_{1,r}(q)$.

Furthermore assume that $n+1 = 2m \geq 4$ and equip V with a nonsingular alternating bilinear form $\langle -, - \rangle$. Let $\mathcal{I}_r$ denote the set of totally isotropic r-dimensional subspaces of V with respect to $\langle -, - \rangle$, where $1 \leq r \leq m$. The *symplectic polar space* $\mathrm{Sp}(2m, q)$ is the geometry with flats $\mathcal{I}_r$, $1 \leq r \leq m$. We will discuss recent results on the p-ranks of the incidence matrices between points and flats of $\mathrm{Sp}(2m, q)$.

The results discussed here are mainly from the joint work of David B. Chandler, Peter Sin and myself.