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Symmetric Bush-type Hadamard Matrices of Order $4m^4$ Exist for All Odd m

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Let n be a positive integer and let J_{2n} denote the matrix of order $2n$ with all entries being ones. A Hadamard matrix $H = (H_{ij})$ of order $4n^2$, where H_{ij} are $2n \times 2n$ block matrices, is said to be of *Bush-type* if

$$H_{ii} = J_{2n}, \text{and } H_{ij}J_{2n} = J_{2n}H_{ij} = 0,$$

for $i \neq j, 1 \leq i, j \leq 2n$.

While it is relatively easy to construct Bush-type Hadamard matrices of order $4n^2$ for all even n for which a Hadamard matrix of order $2n$ exists, it is not easy to decide whether such matrices of order $4n^2$ exist if $n > 1$ is an odd integer. In a recent survey, Jungnickel and Kharaghani wrote "Bushtype Hadamard matrices of order $4n^2$, where n is odd, seem pretty hard to construct. Examples are known for $n = 3, n = 5$, and $n = 9$; all other cases are open".

In this talk, we will show that a construction of reversible Hadamard difference sets by R. M. Wilson and Q. Xiang can be used to construct symmetric Bush-type Hadamard matrices of order $4m^4$ for all odd m.