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The Spectrum of the Hodge-de Rham Laplacian

(4 Lectures)

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We study the relationship between the geometry and the topology of a compact Riemannian manifold (M, g) through the spectrum of the Laplacian acting on p-forms, which is known to be discrete, with a particular interest in the first non-zero eigenvalue. Classically, as regards functions, the goal is to estimate the spectrum with the Riemannian invariants of the manifold, in particular curvature, diameter, and volume. However, for p-forms, the importance of the topology of the manifold appears clearly (recall that the multiplicity of the eigenvalue 0 corresponds to the p^{th} Betti number of M and is a topological invariant), in particular via the collapsing. The goal of these lectures is to give an introduction to this subject and also to give some recent developments and open questions.

Course Plan

- Presentation of the problem. The case of function on domains and on compact Riemannian manifolds, with some classical results.
- Examples for the p-form spectrum: the case of collapsing manifolds and related topics.
- How to get some lower bound on the p-form spectrum. Some recent developments and open questions.