CIMPA-UNESCO-IRAN School on Recent Topics in Geometric Analysis, May 20-June2, 2006, IPM, Tehran

Spectral Geometry

(4 Lectures)

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The first aim of this course is to familiarize the audience with the spectrum of the Laplace operator on Riemannian manifolds, especially in order to supply a necessary background for the other courses of the school (in particular, the courses of Carron and Colbois). Hence, the first part of the course will consists on:

- Basic facts about the spectrum of the Laplacian including discretness and finite degeneracy in the compact case.
- Survey of classical results (Lichnerowicz-Obata, Cheng, Cheeger, Buser) on relationships between eigenvalues and other geometric invariants (curvature, diameter, injectivity radius, isoperimetric constants etc.)
- Discussion of isoperimetric problems related to the eigenvalues of Laplacian : Bergers work and results of Hersch, Li and Yau, Yang and Yau, El Soufi and Ilias, ...

In the last part of the course, we will discuss some recent results and open problems about extremal geometries for the eigenvalues of the Laplacian.