CIMPA-UNESCO-IRAN School on Recent Topics in Geometric Analysis,
May 20-June2, 2006, IPM, Tehran

The Classifying Space of Polarized Hodge Structures

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In the first lecture we recall the Hodge structures on the de Rham cohomologies of compact kähler manifolds and the associate polarizations. We construct the classifying space of polarized Hodge structures D, called the Griffiths domain, and the action of an arithmetic group G_Z on D from the left. We recall Ehresmann's fibration theorem and then the fact that the period maps form coefficient spaces to $G_Z\backslash D$ are holomorphic and satisfy the so called Griffiths transversality. We state the Borel-Baily theorem on the unique algebraic structure of quotients of symmetric Hermitian domains by discrete arithmetic groups. Since except in a few cases D is not symmetric Hermitian, one cannot apply this theorem to D. We will mention such few cases which give origin to the notion of Shimura varieties in algebraic geometry. The partial compactifications of $G_Z\backslash D$ using the ideas of Borel, Baily and many other people after them will be explained.