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Period Mapping and Griffiths Transversality Theorem

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In the second lecture we start to construct a complex manifold P over the Griffiths domain and we extend many notions of the previous lecture, like the action of the arithmetic group, period map, Griffiths transversality and so on to this new complex manifold. In this new context, $L := G_Z\backslash P$ is a complex manifold and enjoys a canonical action of an algebraic group G_0 from the right. The question is whether L has a natural algebraic structure in such a way that the action of G_0 becomes algebraic and moreover every period map becomes a morphism of algebraic varieties. To this end, we recall the generalization of Eisenstein series by Borel and Baily and then we construct their counterparts for P. In the simplest case of Hodge structures arising from elliptic curves we show that such Eisenstein series are not sufficient to algebralize L. The missing function in this case cannot be written as a Poincaré series and it is the Eisenstein series of weight 2. We will introduce such kind of functions for P and explain the fact that they are not automorphic forms and hence they cannot be interpreted as sections of positive line bundles. However, one needs them for possible algebralizations of L.