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Modular Foliations and the Loci of Hodge Cycles

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Through the period map the complex manifold L corresponds to the coefficient space T of a family of varieties with the same topological structure. Despite the fact that in the second lecture we left the question of a reasonable algebraic structure for L without answer, we will start to investigate some equivalent objects in both T and L . One of them is the notion of a modular foliation which does not exist in the classical Griffiths domain. We will recall the Gauss-Manin connection associated to fibrations and then using it we will construct the defining equations of modular foliations. For a 3-parameter family of elliptic curves we will show that such a foliation is given by the so called Ramanujan relations. Some arithmetic properties of modular foliations will be explained. Other objects are Humbert varieties and loci of Hodge cycles. Both are well-understood in L but few is known about their structures in the coefficient space. For instance, a theorem of Cattani-Deligne- Kaplan states that the loci of Hodge cycles is always an algebraic variety. At the end we will show that such varieties are invariant by certain modular foliations.