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Recent Research on Graph Energy

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Let G be a graph on n vertices, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ be its eigenvalues. Then the *energy* of G is defined as

$$E = E(G) = \sum_{i=1}^{n} |\lambda_i| \ . \tag{1}$$

This concept was proposed quite some time ago in the paper [1] (and later on several other occasions). The name was chosen because for some special graphs that represent certain types of molecules, E is related to a part of their electron energy. However, the purpose of the definition (1) is to consider E as just a spectrum–based graph invariant, and to extend its study to all graphs. After a long latent period, the definition (1) attracted the attention of a large number of colleagues, and E became a popular topic of research, especially among mathematicians. Since the year 2000, over 80 papers were produced on graph energy.

In this lecture a survey of main results recently obtained on graph energy will be given, and some open problems pointed out. These results can be classified as follows.

Finding graphs with extremal energy.

Example: Let P_n and C_n be the n-vertex path and cycle, respectively. Denote by $C_6{:}P_{n-6}(i)$ the graph obtained by joining a vertex of C_6 with the i-th vertex of P_{n-6}. Then, among n-vertex unicyclic bipartite graphs, either C_n or $C_6{:}P_{n-6}(1)$ has maximal energy [2, 3]. Among n-vertex unicyclic bipartite graphs, different from C_n and $C_6{:}P_{n-6}(1)$, the graph $C_6{:}P_{n-6}(3)$ has maximal energy [4].

Finding equienergetic graphs

Example: Let $L(G)$ be the line graph of the graph G, and $L^2(G) = L(L(G))$. If G_1 and G_2 are two n-vertex regular graphs of degree r , $r \geq 3$, then $E(L^2(G_1)) = E(L^2(G_2))$ [5].

Bounds for energy

Example: Let G be a regular n-vertex graph of degree r , $r \geq 1$. Then $E(G) \geq n$. Equality holds if and only if every component of G is a complete bipartite graph [6].

References

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