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Relations on Some Topological Indices of a Graph

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Topological indices of graphs have been studied in special cases. In this work we are going to find some relations between topological indices of a graph and some relations between indices and other graph parameters in general.

Let $G = (V, E)$ be a graph with vertex set V and edge set E. $W(G)$, the *Wiener index* of G is defined as the sum of the distances between each pairs of vertices in graph. More precisely $W(G) = \frac{1}{2}\sum_{u,v\in V} d(u, v)$, where $d(u, v)$ is the distance between vertices u and v. $MTI(G)$, *Schultz index* of G is defined to be $\sum_{i=1}^{n}\sum_{j=1}^{n} d_i(A_{ij} + D_{ij})$, where d_i is the degree of vertex i in $V(G)$, $A = [A_{ij}]$ is the adjacency matrix and $D = [D_{ij}]$ is the distance matrix of G. All the graphs here are connected. The maximum and minimum degree of the vertices of G denoted by Δ and δ, respectively. Also n and e are the number of the vertices and edges of G, respectively.

Our first result is an inequality between Wiener and Schultz indices:

$$2\delta(e + W(G)) \leq MTI(G) \leq 2\Delta(e + W(G)). \tag{1}$$

Then we look for description of indices using other well-known graph parameters. We describe indices in terms of *dominating sets* and *independent sets* by finding some useful relations.

Let $\{v_i\}, (1 \leq i \leq n)$ be the vertex sequence of graph G, $S = \{v_1, ..., v_\gamma\}$ a minimum dominating set and $d_{ij}(1 \leq i, j \leq n)$ distance of vertex v_i from v_j. We can take subsets $V_1, ..., V_\gamma$ of V such that V_i is dominated by v_i (except for v_i itself) and $V_i \cap V_j = \emptyset$, $(1 \leq i, j \leq \gamma)$. Now, we prove that

$$W(G) \leq (n - \gamma) + \sum d_{ij}(1 + |V_i| + |V_i||V_j|), \tag{2}$$

where γ is the domination number of G.

Also we prove that

$$W(G) \geq 2\binom{\alpha}{2} + \binom{n-\alpha}{2} + \frac{\alpha(n-\alpha)}{2}, \tag{3}$$

where α is the independence number of G.