IPM Institute for Studies in Theoretical Physics and Mathematics

DAILY NEWS

Tuesday, April 24, 2007, Fourth Issue

We asked …

We asked our distinguished guests the following two questions:

- **In your opinion what is the most important theorem in Algebraic Graph Theory?**
- **In your opinion what is the most important conjecture in Algebraic Graph Theory?**

They responded …

Richard Brualdi

- The following is one of my favorites. It has no purely combinatorial proof; all proofs use some ideas of Linear Algebra. To decompose the complete graph K_n into complete bipartite graphs, at least $n-1$ graphs are required (The Graham Pollak Theorem).

- For every field F, there exists an odd integer k and $\varepsilon > 0$ such that every matrix A of order n with non-zero on the main diagonal such that the rank $A \leq \varepsilon_n$ contains an odd alternating cycle (in the associated diagraph), (The odd alternating cycle conjecture).

Chris Godsil

- Hoffman-SingletonTheorem: A Moore Graphs of diameter 2 and valency k exists only if $k = 2, 3, 7, 57$
(A Moore graph is a graph with diameter d and girth $2d+1$).

- Bannai and Ito: For each integer $k \geq 3$, there are only finitely many distance-regular graphs of degree k.

Willem Haemers

- - Perron-FrobeniusTheorem on nonnegative matrices.
- Classification of all graphs with smallest eigenvalue at least -2 by Cameron, Goethals, Seidel and Shult.

- Hadamard matrices exist for all orders divisible by 4.

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Steve Kirkland

- I'm an old fashioned guy, so I'll go with the Matrix-Tree Theorem. Its proof uses some nice ideas from linear algebra to discuss a problem in graph theory, and I find it quite elegant. The Matrix-Tree Theorem also serves as a nice entrée into algebraic graph theory, and so helps to recruit new researchers into the discipline.

- I expect that the most important conjecture has yet to be made! However, a personal favorite is the Brualdi-Li conjecture, which is concerned with the structure of the tournament on $n = 2m$ vertices whose adjacency matrix has maximum spectral radius (over the class of tournaments on n vertices). The conjectured maximizer has a deceptively simple structure, but the conjecture itself has resisted solution for over 25 years.

Elena V. Konstantinova

- I think that some results in the classification of distance-regular/distance-transitive graphs are very important in Algebraic Graph Theory.

- Every connected Cayley graph on a finite group has a Hamiltonian cycle.

Jack Koolen

- The result by Cameron, Goethals, Seidel, Shult which says that every regular graph with smallest eigen value -2 is either a Cocktail Party graph, a line graph or has at most 28 vertices.

- For given $k \geq 3$,there are finitely many distance-regular graphs with valency k (This is known as the Bannai-Ito Conjecture).

László Lovász

- Colin de Verdièrès spectral characterization of planar graphs.

- Let $A=(a_{ij})_{ij=1}^{n}$, where

$$a_{ij} = \begin{cases} 1 & \text{if } i|j \text{ or } j = 1 \\ 0 & \text{otherwise} \end{cases}$$

Then $\det(A) = O(\sqrt{n})$.

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Bojan Mohar

- I do not think that there is the most important theorem in this area, although there are many important results. My favorite one is the "Lovasz Sandwich Theorem" involving the clique number, the chromatic number and the Lovasz θ-function.

- My favorite conjecture may be the one claiming that all vertex-transitive graphs, with finitely many exceptions, are Hamiltonian. However, I do not think that this is an important problem.

Peter Rowlinson

- Interlacing theorem for eigenvalues.

- There is no Moore graph of degree 57.

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Daily Program

9:00-10:00	10:00-10:30	10:30-11:00	11:00-12:00	12:00-14:00	
R. Brualdi (2)	A. El Sahili	Coffee Break	S. Akbari	**Group Photo**	Lunch
14:00-18:00	19:00-22:00				
City Excursion	**Conference Dinner**				

Chairman of the morning session: Bojan Mohar

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From Mathematical Apocrypha

Those who hung out with Erdös were accustomed to the activity of "Uncle Paul Sitting." Paul was usually quite helpless with most things that were not mathematics. This was perhaps a product of his upbringing. His parents were overly protective; indeed, Paul did not butter his own bread until, at the age of 21, he was on a trip to England. When he visited a university, he expected people to fetch envelopes and tablets and pens for him, make phone calls, give him rides, wash his clothes, feed him, and take care of all his other needs. Erdös did seem, at one point in his life, to have an amorous liaison—with a woman. After a time it became apparent to her that Paul really valued her because she gave him rides to wherever he needed to go. End of relationship.

Erdös could remember two dozen phone numbers at a glance, but appeared to be absent-minded about many things. Once, when visiting CalTech, he lost his sweater twice in the same day. The first time it was recovered and the second time not. When he visited my own university we asked him for his Social Security number so that we could pay him. He did not know it. At dinner, since no mathematics was being discussed, he fell asleep.

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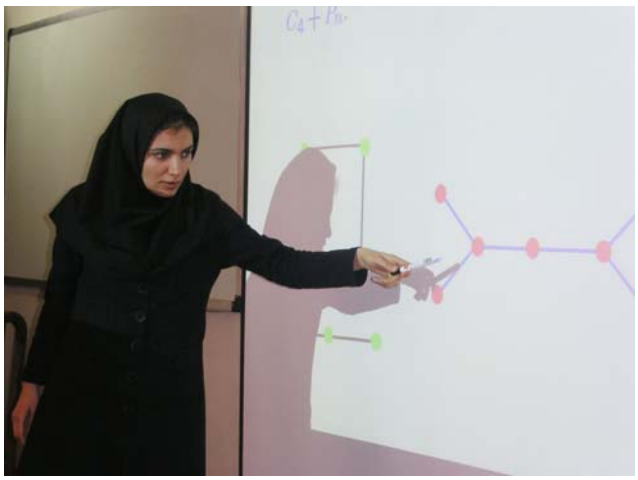

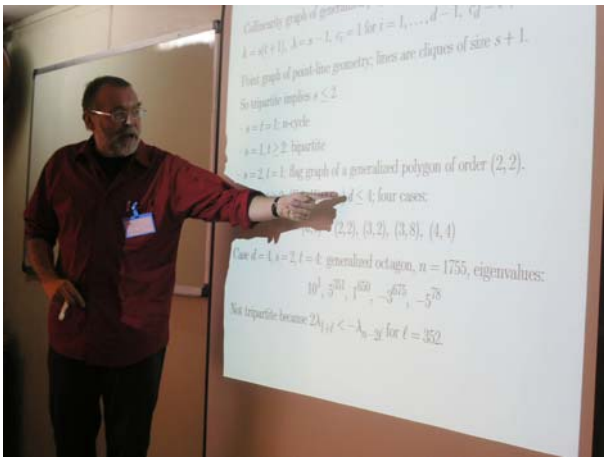
Collinearity graph of
$k = s(t+1)$, $\lambda = s-1$, $c_i = 1$ for $i = 1, \ldots, d-1$, c_d
Point graph of point-line geometry; lines are cliques of size $s+1$.
So tripartite implies $s \leq 2$.
$s = t = 1$: 6-cycle
$s = 1, t \geq 2$: bipartite
$s = 2, t = 1$: flag graph of a generalized polygon of order $(2,2)$.
$d \leq 4$; four cases:
$(2,2)$, $(3,2)$, $(3,8)$, $(4,4)$
Case $d = 4$, $s = 2$, $t = 4$: generalized octagon, $n = 1755$, eigenvalues:
$10^1, 5^{704}, 1^{650}, -\sqrt{675}, -5^{78}$
Not tripartite because $2\lambda_{1+t} < -\lambda_{n-2t}$ for $t = 352$.